HUMAN FACTORS IN FIELD EXPERIMENTATION DESIGN AND ANALYSIS OF AN ANALYTICAL SUPPRESSION MODEL

Michael Peter Mueller

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

Human Factors in Field Experimentation
Design and Analysis of an
Analytical Suppression Model

by

Michael Peter Mueller
and
Karl-Heinz Pietsch

September 1978

Thesis Co-Advisors: D. P. Gaver
G. K. Poock

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The result for the modelling approach to suppression indicates source dependences on quantitative as well as on qualitative features.

The functions are left quite general, although some functional forms are derived and discussed.

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by

Michael Peter Mueller
Captain Federal German Army
Ingenieur (graduiert), 1974, Technical Military Academy, West-Germany

Karl-Heinz Pietsch
Major Federal German Army
Betriebswirt (graduiert), 1971, Technical Military Academy, West-Germany

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

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NAVAL POSTGRADUATE SCHOOL
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I. FORMULATION OF THE PROBLEM

A. INTRODUCTION

In recent years considerations of Human Factors aspects in the military has gained more and more importance. The study of psychological, physiological, and environmental conditions and their influences on the performance of men in "Man-Machine-Systems" supports the development of new doctrines, design of weapon systems as well as training programs for troops.

One phenomenon in military man-machine-systems is suppression. Modelling this phenomenon has only recently been given much attention.

This is partly because modellers did not understand exactly neither the causes of suppression nor how it affects the course of combats.

There is an intuitive feeling that when a soldier or a combat unit is being fired upon, it will be less effective than when it is not receiving fire. This is generally referred to as suppression, but it can include much more. In the broadest sense, suppression can effect individuals, units, or weapon systems in different types of combat.

This paper will limit itself to individuals or small infantry units.

In order to be able to build up functional relationships based on the suppression idea, between different categories of people and time, this paper will first examine different rates as input to analytical Lanchester-type-models. The rates will be based on stochastic aspects. The modelling is done under different viewpoints; their results will be compared and discussed.

In the second part of the thesis, the parameters of the models will be tested in a regression analysis against real world data which were placed at the writers' disposal by CDEC.

The goal of this paper is not to present a final framework of suppression; however it may contribute to clarification of aspects of suppression, and help to embed it as a component in future large size models.

B. BACKGROUND

The strains imposed on individuals in our society are constantly increasing. Modern technologies and constant efforts for improvements of standards of living lead to growing difficulties in adjustments or sometimes to complete failures to adjust. These facts create stress and we may observe that the degree of stress increases with the difficulty of the adjustment-problem.

The term stress will be used as a substitute for what might be called otherwise as anxiety, conflict, emotional distress, extreme environmental conditions, ego-threat, frustration, threat to security, tension or probably arousal. Stress can be thought of as the result of almost any environmental interference [Appley, p. 2].

The stress-generating features of the civilian environment are great, but the environment created by modern warfare possesses additional features which result in an increase of stress. The combat environment created by the weapon power of the enemy causes a constant threat to life. The soldier has to operate under this threat and naturally he will respond with constantly recurring fear. This may break down the soldier's psychological and physiological resistance.

Fear and anxiety in battle is common, being experienced by between 80 and 90 % of combatants. Pains in the stomach, fatique, dizziness, perspiration, and enhanced heart-beat are some vegetative correlates to fear and anxiety. Of course the moment when the individual soldier reaches his breaking point varies and depends on individual psychological and physiological resistance and the severity of the battle.

One interesting observation from Agrell was that auditory sensations convey the stress of battle most strongly and most directly. The psychological effect of weapons is directly related to their sound level and the frequency with which the sound occurs [Agrell,p.215].

Each enemy grenade causes the soldier to react constantly with fear. Stress and fear can have a significant sensory operating characteristic, e. g. the detection threshold and/or sensitivity may decrease as a result of stress [Weltmann, G, Christianson, R.A. and Egstrom, G.H., p.423-430].

C. MILITARY SUPPRESSION IN COMBAT-ENVIRONMENT

When weapons are used in combat, there are two types of effects that they have. The first type of effect is physical damage or injury to the target and the second type of effect is psychological.

This second effect of a weapon has led to the term or concept "Combat Suppression". There exist a general belief that fire suppression is important, but the importance of suppression effect on combat outcomes as compared to the effect of other areas such as firepower, mobility, intelligence, command/control has not been quantified adequately.

1. Definitions

> Suppression can be generally defined as the temporary degradation in the quality of performance of an individual soldier or unit by an internal or external stimulus.
>
> CDEC, 1977

A more useful operational definition in terms of performance capability changes is provided by the "Report of the Army Scientific Advisory panel Ad Hoc Group on Fire Suppression".

> Fire suppression is a process which causes temporary changes in performance capabilities of the suppressee (suppressed system) from those expected when functioning in an environment he knows to be passive. These changes are caused by signals from delivered fire or the threat of delivered fire, and they result from behaviors that are intended to lessen risk to the suppressee.
>
> Ad Hoc Group, 1976

This definition emphasizes that suppression is not a

single effect which can be measured totally on a single quantitive scale. Suppression effects are multidimensional and the "amount of suppression" varies among these dimensions (e.g. fire impact points, soldier's characterists, and reaction to the fire, his combat experience etc.).

Suppressive fire in a combat environment can suppress a number of combat activities; for example: firing, search for and observation of targets, movements of units or command and control.

2. Structure Of Fire Suppression Process

Fire suppression is a complicated process involving many physical, environmental, physiological, behavioral, and operational variables. The important point to emphasize is that the behavior involved is in response to stimuli that originate both externally (combat environment) and internally (personal background, training and experience) to the soldier suppressee. These aspects, however, are not included in this paper.

The intensity and duration of suppression can not be predicted from a knowledge of the combat environment alone. It requires an analysis of the underlying motivational and cultural factors and of the context of the combat environment.

The Fig 1 [Ad Hoc Group, p.36] shows a schematic description of a process when suppressive fire is delivered and its affect on the combat result.

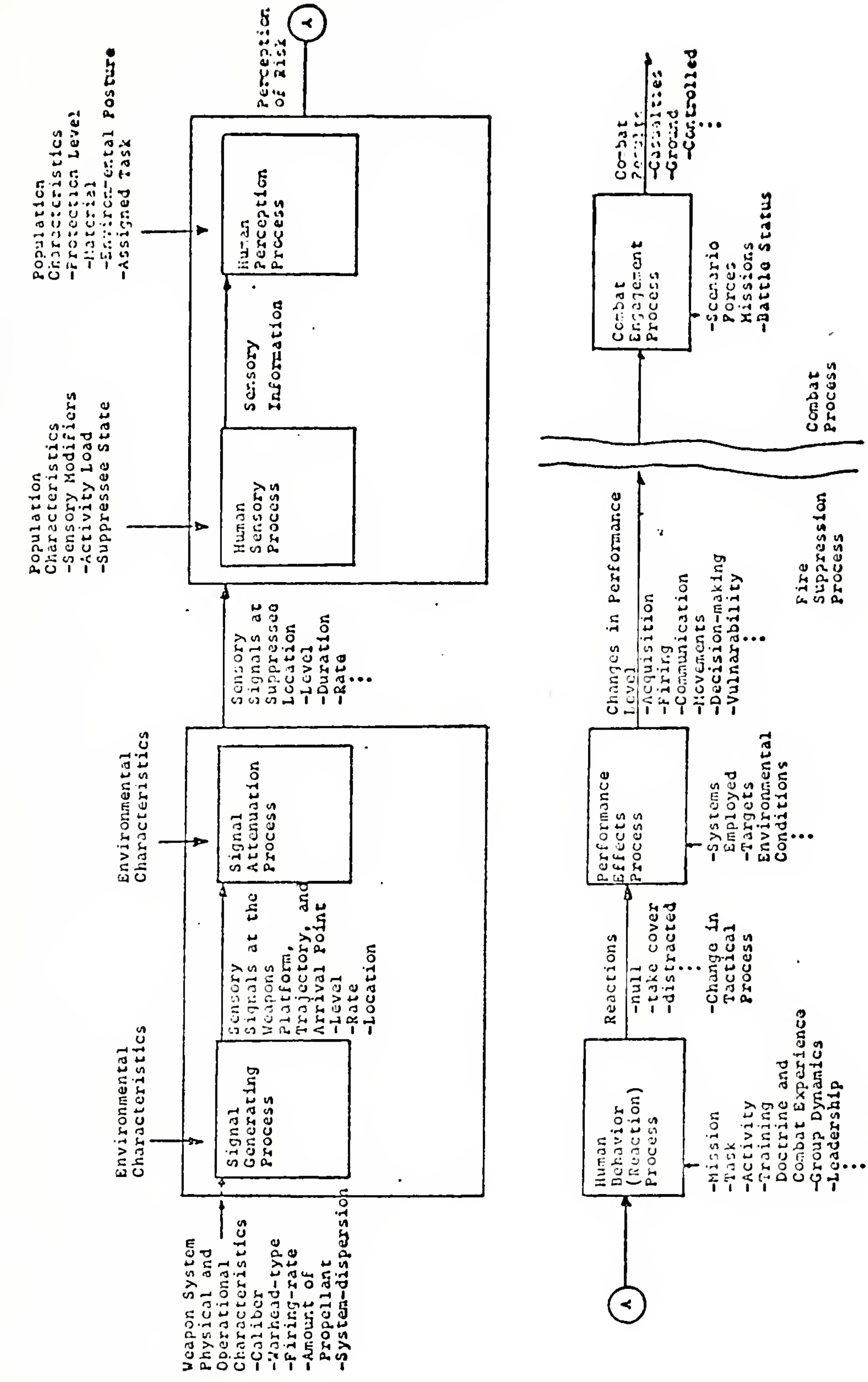


Figure 1 - SCHEMATIC STRUCTURE OF THE FIRE SUPPRESSION PROCESS (Ad Hoc Group, p. 56)

The process is discribed by individual functions.

a. Signal Process

The first process in fire suppression is the generation of signals provided by suppressing weapons.

Inputs to this process are the

characteristics of the weapon systems (caliber, amount of porpellant, warheadtypes etc) and

the environmental characteristics (trajectory, platform, arrival points etc).

Characteristics of the weapon systems can vary in order to increase or decrease suppressive effects.

Some parameters that are considered to be important to suppression signals are:

Muzzle velocity (an increase in muzzle velocity is associated with an increase in signal variables);

Caliber (as caliber increases, the firing signals and projectile signals increase along with lethality);

Projectile weight (penetration depends on weight and velocity at impact and increases shock coupling to ground);

Warhead charge weight (the explosive charge weight determines the energy in the pressure pulse);

Additional parameters like fire frequency and proximity of shots could also be mentioned.

Environmental characteristics are also variables but they can not be determined completely.

Environment has an influence on the signal

generation -and transmission-process. For example, auditory signals that result from the impact of projectiles depend heavily on the nature of the object or material impacted.

A soft yielding material such as dusty ground or sand receiving the impact of a projectile will produce a different pulse and sound than will hard unyielding ground under the same impact.

Sound signals can be attenuated by the shadowing effect of large obstacles or may be increased by echo or reverberation. Visual signals are strongly modified by the condition of lighting. Haze, fog, rain, and snow act similarly to smoke and dust. The visual field is also reduced and interrupted by terrain and other obstacles.

The environment modifies the produced stimuli when they are transmitted to the location of the suppressee. Outputs of this process are the attenuated sensory signals that become imputs to the human sensory receptors.

b. Human Process

Many of the determinants of the soldier's performance on the battle field are unkown or at least uncertain - thought of as influenced by chance factors. This emphasizes the difficulties of predicting human behavior in a combat environment. The human process (sensory and perception) converts the received signals into a perception of the risk.

Battlefield stimuli effecting the individual are detected and converted into sensory data by a process such as vision and audition, so the sensory process suggests that the weapon systems stimuli relevant to suppression are the

-loudness, and
-visual impact.

There exist moderating factors that influence the operating characteristcs of the sensory process and that determines which stimuli are effective.

Sensory modifiers (i.e. earplugs, night-vision-devices) serve to change users' sensitivity range. A major effect of these devices is to change the salience of stimuli.

High concentration on an activity or a high level of effort on an activity (e.g. missile-gunner is tracking a target or reloading his system) may increase the absolute threshold.

The posture of a soldier (standing or sitting) and the sequence of posture (observing, ducking, observing) influences the sensory capabilities (e.g. observing for 10 seconds continuously is not equivalent to observing 5 seconds , ducking 10 seconds and observing 5 seconds) .

The **perception process** integrates sensory and other information into a perception of the risk. Risk refers to the uncertainty of damage, injury, or loss. It characterizes decision situations in which the consequences of choosing an action are uncertain.

If there is no uncertainty in the possible outcome, there is no risk.

Perceived risk is a function of uncertainty and the subjective value the individual associates with each outcome.

Perceived risk represents the output of the combined sensory and perception process. It depends on the individual's experience and training in assessing risk from sensory information. Also cover provided by the environment and the individual's posture may influence risk perception.

c. Reaction Process

Given the input perception of risk, this process causes physical and mental reactions, which depend on the

- current mission
- task
- activity

-combat training doctrine and experience
-group dynamics and
-the quality of leadership.

It is conjectured that two individuals who perceive the same degree of high risk , but who have different amounts of combat engagement experience, might be likely to react differently to the risks.

The soldier's reaction is also influenced by his current state. A soldier who has recently ducked may be more likely to duck than one who has not , given the same delivered fire.

Prior reaction or sequence of reactions may be a good predictor of the coming reaction.

d. Performance Effects Process

Given the reactions of the human behavior process, it is conjectured that these directly affect the performance of certain activities of the suppressee in a calculable way.

If for example the suppressee takes cover, he may fire less often and less accurately and also might be less vulnerable. The magnitude and duration of these changes in performance are dependent on the characteristics of the system employed by the suppressee and the target of his activity.

So the nature and duration of change in performance capabilities is determined by the performance effects process.

3. <u>Suppression In Field Experimentations</u>

A fire suppression research program requires significant experimentation on behavioral attitudes and reactions to risk. This necessity causes tremendous difficulties in trying to induce actual behavior in soldiers

in field experiments. Former studies showed, that the soldiers felt true psychological stress only in situations in which they believed that they were in real danger. Such situations are difficult to contrive and to control. Social and ethical limits and legal regulations preclude the introduction of actual physical risk. Soldiers must be taught the "rules and risk" defined in that context. The success of playing the role, being an individual participating in a combat engagement, depends on the soldiers' motivation and willingness.

Because of these reasons and the multidimensional shape of the fire suppression process as mentioned earlier, suppression in field experimentations may be restricted only to some variables involved in this process.

D. APPROPRIATE OBJECTIVES

The overall objective of a fire suppression research should be to relate changes in performance capabilities caused by fire suppression [Ad Hoc Group, p. 110]. Responding more directly the following objectives may be determined:

Indicating the effects of suppression on combat results, i. e. to develop rates of suppression. These values may be compared to other effected areas and probably employed in computer simulations. These numbers represent two kinds of variables: Weapon system variables and human suppression performance, given operational and environmental conditions.

Determining characteristics of suppressive fire systems, characteristics which should be assigned to such a weapon system. Results are developed experimentally . Chapter III B. will display some evaluated parameters and constants for the developed

model, which may also represent the suppressive characteristics of the used weapon systems.

Reducing suppressive effects. Ways to reduce the effect of suppressive fire may also be considered as an appropriate objective of suppression research. Special training or equipment can be assigned to the soldiers or new tactics can be developed. This objective is beyond of the research of this paper.

E. POSSIBLE ALTERNATIVES

In order to get information about the fire suppression process, previous investigations were based on interviews and questionaires, because valuable information of the fire suppression process is stored in the minds of combat veterans. Studies on veterans of the Vietnam conflict and the wars of the Near East would be especially useful, since newer weapons were employed and the combats were shorter and more intensive.

These studies may provide a good insight to the suppression process and/or may also deliver valuable inputs for the modelling approach.

II. CONSTRUCTION OF AN ANALYTICAL MODEL

A. ASSUMPTIONS AND SIMPLIFICATIONS

Based on the foregoing discussion, the following model is a detailed model [Taylor,1978,p.12] which starts out by considering the behavior aspect of a human being under the influence of artillery fire power. It is assumed that such simple models, which represent a small part of a total scenario, can be used profitably to investigate system dynamics of more complex models. The value of the model derives from the fact that it forms intuitively plausible and transparent subsets in a large composition of other subsets which determine the basic structure of the complex operational model. In other words, the whole is described in terms of the sum of its parts.

The basic concern of the analytical model developed here will be to model the behavior of an individual experiencing artillery fire, considered as a function of time, where that behavior depends upon ammunition types and location of detonating rounds.

On the basis of particular assumptions and simplifications it will be possible to apply the results obtained to a group of people (a force) on a battelfield. The result of these considerations will provide a relationship between time and the actual number of people affected by the fire power of the artillery . This last step of the model is carried out by using Lanchester type equations, so called after the pioneering work of F. W. Lanchester. Finally, the models enable one to estimate the total firepower of the force at any point in time; depletion

of total firepower is caused by attrition and by suppression.

E. RATES FOR THE MODEL

1. Rates Of Suppression

The basic considerations in the preceeding paragraph support the assumptions, that the behavior of a suppressee can be expressed by a conditional probability of suppression as a function of miss distance r and aspect angle θ. This function is represented by the family of surfaces of the form:

$$P(S/\theta,r) = \exp[-\frac{1}{K} r^2(1 - \varepsilon \cos \theta)] \qquad (2.1)$$

where $P(S/\theta,r)$ is the conditional probability of suppression given that a particular round impacts under a certain aspect angle θ and a certain miss distance r away from the foxhole.

The line along the angle $\theta=0°$ is identical with the line of sight. It is the main direction of observation.

The constant K and ε are parameters, which are determined by the experiment itself and by the environmental conditions.

They can be influenced by factors as discussed in chapter I which may be recalled here briefly.

- type of ammunition
- frequency of arrival of rounds
- perceptual damage
- total time spent in the foxhole (learning process)

- noise appearance of the rounds
- flash light intensity of the rounds
- performance
- personal factors like age, personal condition, etc.
- degree of stress
- motivation

The computational evaluation of the constant K and ε will be performed in chapter III B. and C. In particular it will be important to determine K as it varies with different types of ammunition.

The mathematical conditions for the two parameters K and ε are:

$0 < \varepsilon < 1$

$K > 0$

If θ is held fixed $, 0° < \theta < 360°,$ and $P(S/\theta, r)$ varies between $0 < P(S/\theta, r) < 1$ we obtain a family of functions, which is two-dimensional and shows an exponential relationship between $P(S/\theta, r)$ and r.

This is illustrated in the following figure, where the angle θ is held fixed at 0° and 180°. The parameters K and ε are assumed as being 1500 and 0.7 respectively.

r/m = distance/meter

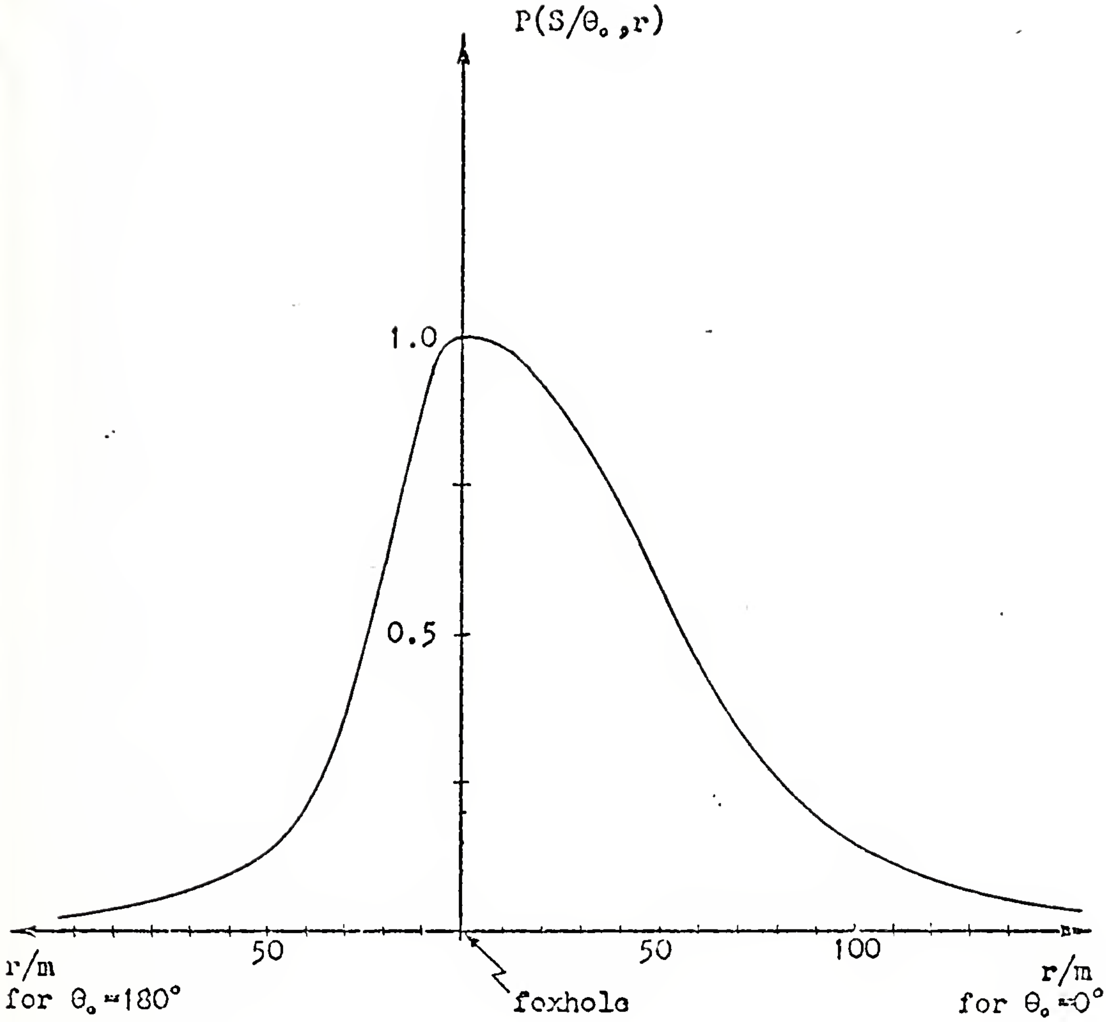


Figure 2 - FUNCTION $P(S/\theta_0,r) = \exp[-\frac{1}{K} r^2(1 - \varepsilon \cos \theta)]$

If on the other hand $P(S/\theta,r)$ is fixed, $0<P(S/\theta,r)<1$, and the equation is solved for r, we will obtain an "egg-shaped" function with iso-levels of probability of suppression $P(S/\theta,r)$. The foxhole is located in the middle of the coordinate system. Along $\theta=0°$, the range r is a max for a certain fixed probability of suppression, while at $\theta=180°$, r is a min for the same probability.

The function takes the form:

$$r^2 = \frac{-K \ln P(S/\theta,r)}{1 - \varepsilon \cos \theta} \qquad (2.2)$$

The following graph shows the family of functions for 4 representative selective probabilities of suppressions.

$$P(S/\theta,r)=0.1 \cdot i \qquad i=1,2,3,4 \qquad (2.3)$$

The parameters K and ε are again assumed to be 1500 and 0.7 respectively.

r/m = distance/meter

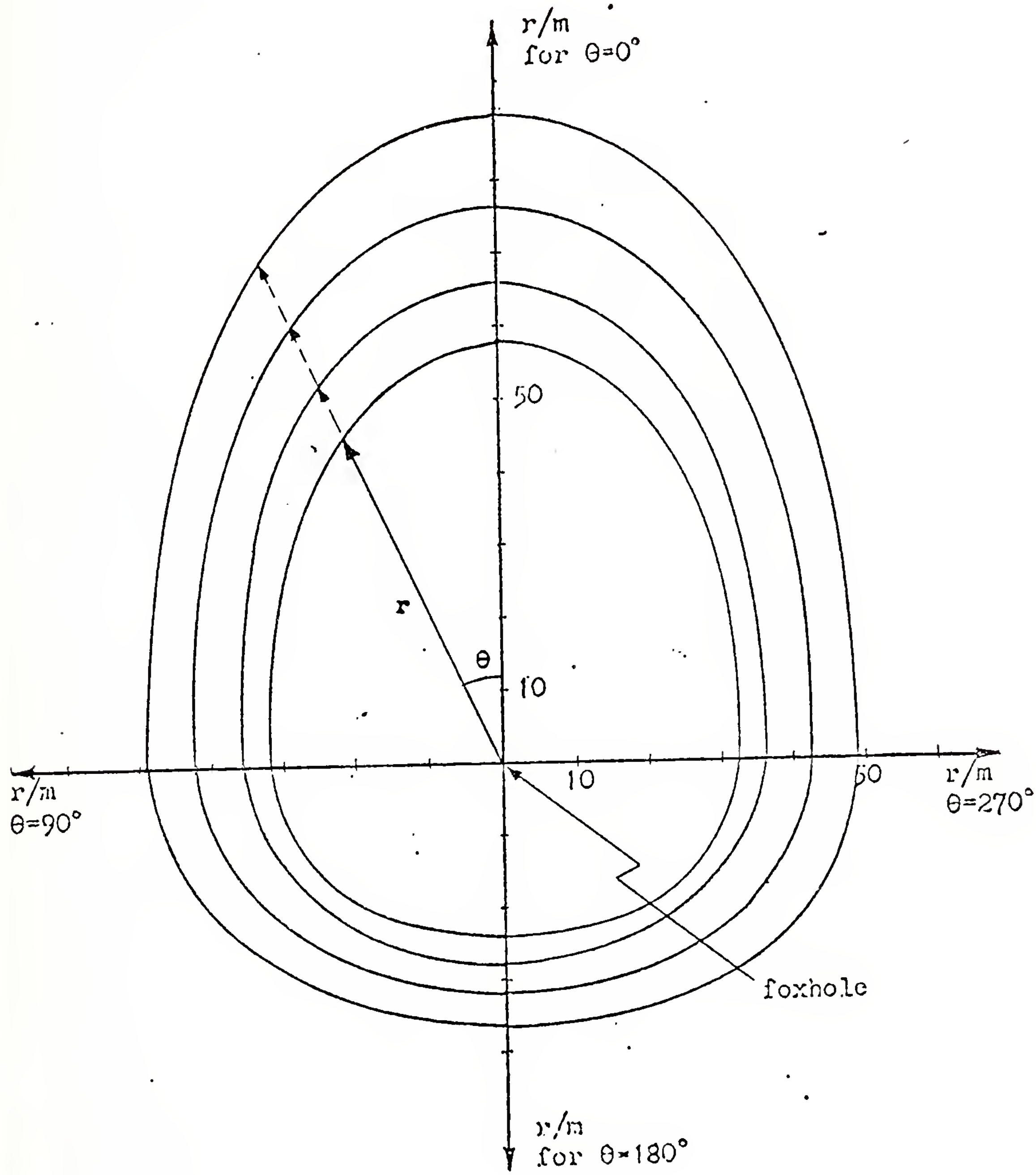


Figure 3 - ISOLINES $r^2 = \frac{-K \ln 0.1 \cdot i}{1-\varepsilon \cos\theta}$, $i = 1,2,3,4.$

These "egg-shaped"-functions of iso-probabilities of suppression simulate the reaction of human beings looking along the main axis $\theta=0°$ in a very simple way. The functions take into account the visual and accoustical perception resulting from any given detonation of a round, where noise and light are the major stimuli.

One could think of other functions wich simulate the behavior of an antitank gunner exposed to artillery rounds for instance:

$$P(S/\theta,r) = L\cdot\exp[-\frac{1}{K}\, r^2(1 - \varepsilon\cos\theta)] \qquad (2.4)$$

This type of function allows the probability of suppression $P(S/\theta,r)$ to be smaller than one at its maximal value and has the same general behavior as the function before. As a third modification:

$$P(S/\theta,r) = \exp[-\frac{1}{K}\, |r|(1 - \varepsilon\cos\theta)] \qquad (2.5)$$

these functions have the disadvantage that they have a discontinuity at r=0. The integration which is necessary in the following derivation is more difficult than the chosen one.

However, the crosscut sections of these functions are not "egg-shaped" iso-functions but rather simple conic sections (ellipses) where the foxhole is located in the center of one focus point.

The paper will continue with the function first described in (2.1), because of its simplicity and variety of application.

In order to derive a rate of suppression for the model, it is necessary to evaluate now, in a second step,

the unconditional probability of suppression P(S) for any incoming round, no matter where it will impact around the foxhole. This will be performed by matching the conditional probability $P(S/\theta,r)$ with the area-hit-probability P(A).

Consequently, to the engagement procedures of the artillery, the targets which shall be suppressed by the artillery are categorized as small personal targets. This implies that the mean point of impact (MPI) of a given set of rounds lies on the target, which means also, that the density has its max value at that point. According to the U.S. Army Field Manual FM 6-161-1 page 2-2 it can be ascertained that the MPI-error is destributed normally with its mean at the aim-point. Thus it will be hypothesised that in such a case the distribution of incoming rounds is bivariate normal with parameters

$$\mu_1 = 0$$

$$\mu_2 = 0 \tag{2.6}$$

$$\sigma_1 = \sigma_2 = \sigma$$

In order to simplify the model, it is proposed that the dispersion of rounds expressed in the standard deviation is equal in all directions. With

$$\rho = 0 \tag{2.7}$$

no correlation is assumed between horizontal and vertical deviation. The normality is also preserved if more than one artillery gun is shooting. The essential change which has to be made when a whole artillery unit will deliver the rounds will be the value of the standard deviation σ

In addition, σ is determined by factors like:

dispersion

the fact that the location of the target is only estimated and an artillery unit delivers rounds in a fire area.

type of ammunition

conditions of the weapon systems (precision)
ballistic properties of the rounds

distance to target

caliber

wind and other weather conditions.

It follows from the bivariate normal density for artillery hits:

$$f(x_1,x_2) = \frac{1}{2\pi(1-\rho^2)\sigma^2} \tag{2.8}$$

$$\times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-r_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x_1-r_1}{\sigma_1}\right)\left(\frac{x_2-r_2}{\sigma_2}\right) + \left(\frac{x_2-r_2}{\sigma_2}\right)^2\right]\right\}$$

and by inserting the previous mentioned assumptions and changing to polar coordinates:

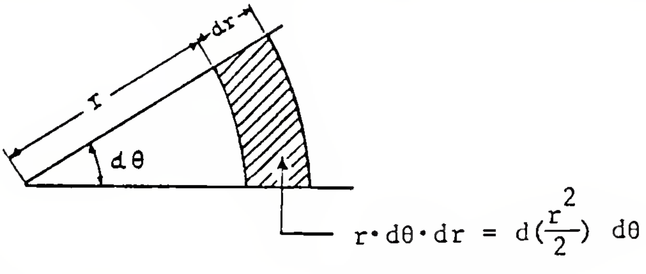
$$r^2 = x_1^2 + x_2^2$$

$$\cos\theta = \frac{x_1}{r} \tag{2.9}$$

that the unconditional probability of suppression P(S) can be written in the form:

$$P(S) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\infty} P(S/\theta,r)\cdot f(r,\theta)\ d\theta\ r\,dr \tag{2.10}$$

Recall that an area element in polar coordinates can be expressed:



The conditional probability of suppression is

$$P(S/\theta,r) = \exp[-\frac{1}{K}\ r^2(1-\cos\theta)] \tag{2.11}$$

and the density for artillery hits $f(r,\theta)$ is

$$f(r,\theta) = \frac{1}{2\pi\sigma^2} \exp[- \frac{1}{2} (\frac{r}{\sigma})^2] \tag{2.12}$$

inserting both into the expression above, we can perform the integration:

$$P(S) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\infty} \exp[- \frac{1}{K} r^2 (1 - \varepsilon \cos\theta) \frac{1}{2\pi\sigma^2} \exp[- \frac{1}{2}(\frac{r}{\sigma})^2] \cdot d(\frac{r^2}{2})\, d\theta \tag{2,13}$$

After a change of integration variable;

$$z_{r=0} = 0 \qquad \frac{r^2}{2} = z$$

$$d(\frac{r^2}{2}) = dz \qquad z_{r\to\infty} = \infty \tag{2.14}$$

we find

$$P(S) = \int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z=\infty} \exp\left[-\left(\frac{2}{K}(1 - \varepsilon\cos\theta) + \frac{1}{\sigma^2}\right)\right] \cdot \frac{1}{2\pi\sigma^2}\, dz\, d\theta \tag{2.15}$$

$$P(S) = \int_{\theta=0}^{\theta=2\pi} \frac{1}{-2\pi\sigma^2 \left(\frac{2}{K}(1 - \varepsilon\cos\theta) + \frac{1}{\sigma^2}\right)} \cdot \exp\left[-\left(\frac{2}{K}(1 - \varepsilon\cos\sigma) + \frac{1}{\sigma^2}\right) \cdot z\right]\Bigg|_{z=C}^{z=\infty} d\theta$$

$$P(S) = \int_{\theta=0}^{\theta=2\pi} \frac{1}{2\pi\sigma^2\left(\frac{2}{K} - \frac{2\varepsilon}{K}\cos\theta + \frac{2K}{2K\sigma^2}\right)} \cdot d\theta$$

$$P(S) = \frac{K}{4\pi\sigma^2} \int_{\theta=0}^{\theta=2\pi} \frac{1}{\left(1 + \frac{K}{2\sigma^2}\right) + (-\varepsilon)\cos\theta} \cdot d\theta \tag{2.16}$$

by replacing

$$b = 1 + \frac{K}{2\sigma^2}, \qquad c = -\varepsilon \tag{2.17}$$

the integral reduces to a known form which can be solved.

$$P(S) = \frac{K}{4\pi\sigma^2} \int_{\theta=0}^{\theta=2\pi} \frac{d\theta}{b + c\cdot\cos\theta} \qquad \text{where} \quad b^2 > c^2$$

$$P(S) = \frac{K}{2\pi\sigma^2} \int_{\theta=0}^{\theta=\pi} \frac{d\theta}{b + c\cdot\cos\theta} = \frac{K}{2\pi\sigma^2} \left[\frac{2}{\sqrt{b^2-c^2}} \arctan \frac{(b-c)\tan\frac{\theta}{2}}{\sqrt{b^2-c^2}} \right]_0^{\pi}$$

$$P(S) = \frac{K}{2\pi\sigma^2} \cdot \frac{2}{\sqrt{b^2-c^2}} \cdot \frac{\pi}{2} \implies P(S) = \frac{K}{2\sigma^2\sqrt{b^2-c^2}} \tag{2.18}$$

Inserting back the expressions for b and c, one obtains:

$$P(S) = \frac{K}{2\sigma^2 \sqrt{\left(1 + \frac{K}{2\sigma^2}\right)^2 - \varepsilon^2}} \tag{2.19}$$

After the data analysis in chapter III B. and C. where the parameters K, ε and σ will be determined, it will be possible to evaluate the probability of suppression

P(S) according to the preceeding formula.

This probability adequately models those cases where temporary suppression is the only possible response to a given detonation of a round. It may be useful to expand the model by introducing a second paired outcome at any given impacting round. Until now, we had one paired outcome:

The individual was either suppressed

or not suppressed.

In addition to these we consider a second paired outcome:

The individual is permanently suppressed by being wounded or killed

or not permanently suppressed.

If we focus the attention on modelling the effects under this expansion, it is possible to evaluate a new probalility of suppression P(S) and a probability of kill P(K) by making the following assumptions:

The conditional probability of kill unlike the condtional probability of suppression does not depend on the aspect angle θ.

It can be represented by a smooth curve of the following form:

$$P(K/r) = \exp(-\frac{1}{H} r^2) \qquad (2.20)$$

where H is a positive constant i.e. $H > 0$. This function was selected on intuitive grounds rather then based on real-world data. Taking a vertical cut through the surface function above along any angle θ, one obtains the following figure. The parameter H is arbitrarily chosen as being H=50. Clearly this parameter is a function of different factors like terrain and ammunition.

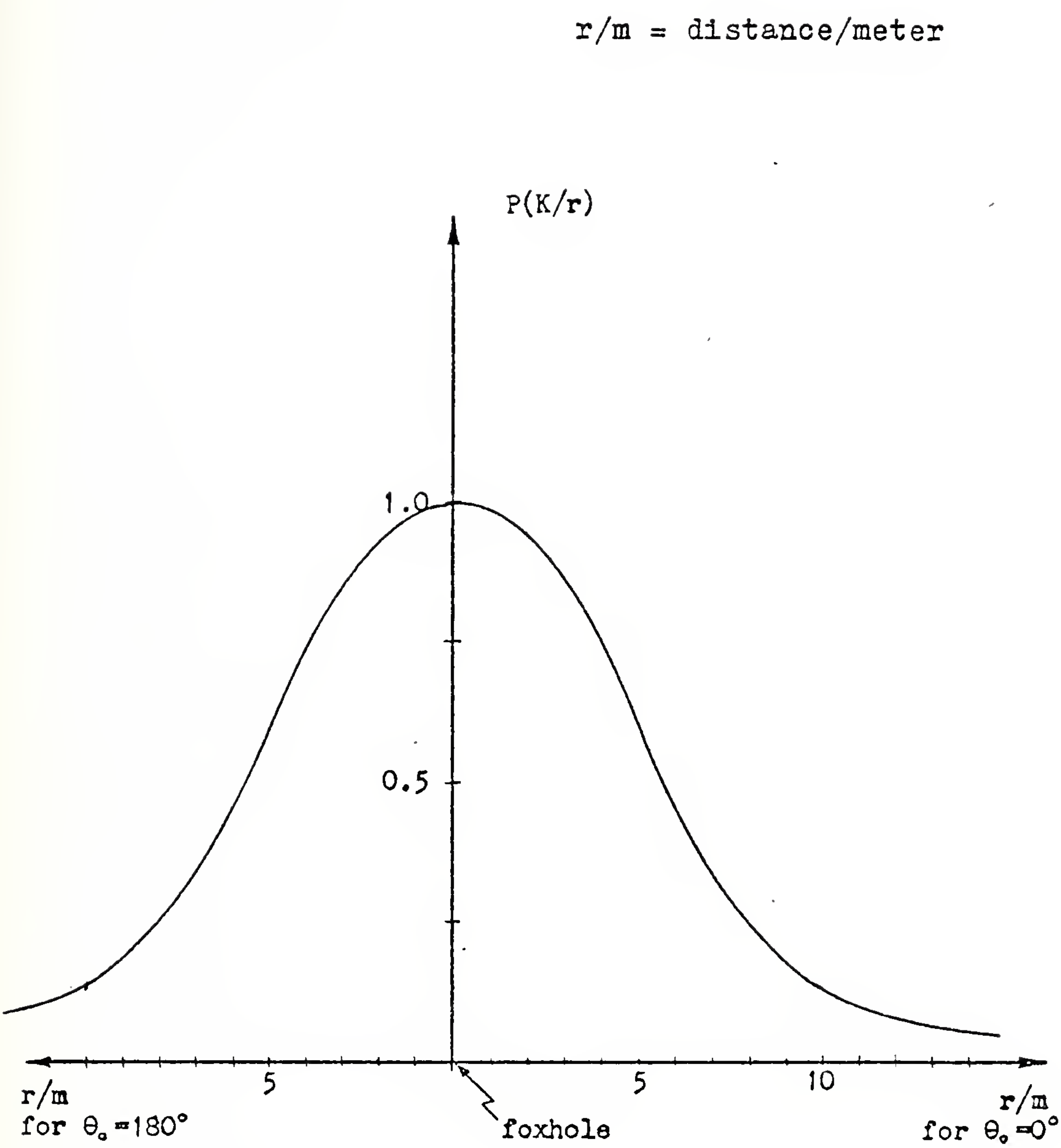


Figure 4 - FUNCTION $P(K/r) = \exp(-\frac{1}{H} r^2)$

If on the other hand $P(K/r)$ is fixed, $0<P(K/r)<1$, and the equation is solved for r, one will obtain concentric circles with iso-levels of probability of kill $P(K/r)$ around the foxhole.

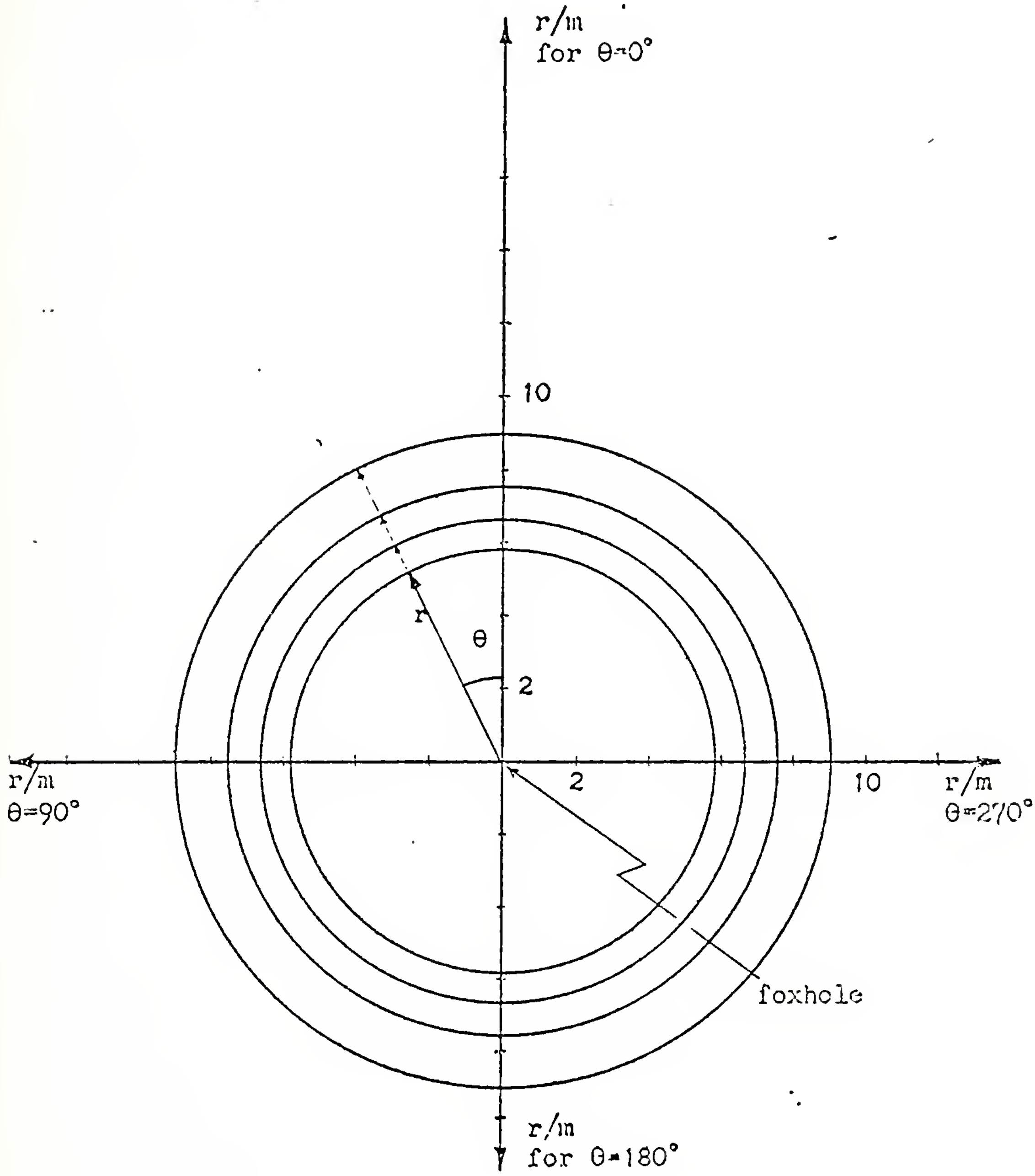
The function has the form:

$$r^2 = - H \cdot \ln P(K/r) \qquad (2.21)$$

The following Fig 5 shows the family of functions for 4 representative selective probabilities of suppressions.

$$P(K/r) = 0.1 \cdot i \qquad i = 1,2,3,4 \qquad (2.22)$$

The parameter H is again chosen to be 50.



r/m = distance/meter

Figure 5 - ISOLINES $r^2 = -H \cdot \ln P(K/r)$

The concentricity around the foxhole expresses the fact that no matter in which direction the person in the hole is looking the kill effect is just determined by the distance. It would be beyond the paper to verify this assumption , and it would be extremely difficult to collect data for it. The reader must be content for the moment with the earlier presented intuitive argument.

This newly introduced function leads to a revision of the probability of suppression P(S) and the evaluation of the unconditional probability of kill, P(K). The density assumed earlier for artillery hits is used again:

$$f(\theta,r) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right]. \qquad (2.12)$$

Probability of suppression $P_K(S)$ but not kill is:

$$P_K(S) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\infty} P(S/\theta,r)[1 - P(K/r)]\cdot f(\theta,r)\ r\ dr\ d\theta \qquad (2.23)$$

Remark: The subscript K at p_K is used to indicate that this suppression probability appears together with the probability of kill P(K).

The expression $P(S/\theta,r)\cdot(1-P(K/r))$ means that a round suppressed but did not kill the individual at (r,θ).

Probability of kill P(K)

$$P(K) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r\to\infty} P(K/r)\cdot f(\theta,r)\ r\ dr\ d\theta \qquad (2.24)$$

The computation for $P_K(S)$ and P(K) runs along similar lines as in the earlier evaluation of P(S). For the specific functions we find:

$$P(K) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\infty} \exp(-\frac{1}{H} r^2) \cdot \frac{1}{2\pi\sigma^2} \exp[-\frac{1}{2} (\frac{r}{\sigma})^2] \, d(\frac{r^2}{2}) \, d\theta \tag{2.25}$$

$$P_K(S) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\infty} \exp[-\frac{1}{K} r^2 (1 - \varepsilon \cos \theta)] \, [1 - \exp(-\frac{1}{H} r^2)]$$

$$\cdot \frac{1}{2\pi\sigma^2} \exp[-\frac{1}{2} (\frac{r}{\sigma})^2] \, d(\frac{r^2}{2}) \, d\theta \tag{2.26}$$

change again the variable of integration

$$\frac{r^2}{2} = z; \qquad z_{r \to \infty} = \infty; \qquad z_{r=0} = 0; \qquad d(\frac{r^2}{2}) = dz \tag{2.14}$$

$$P(K) = \int_{\theta=0}^{\theta=2} \int_{z=0}^{z \to \infty} \frac{1}{2\pi\sigma^2} \exp\left[- (\frac{1}{H} + \frac{1}{\sigma^2}) z \right] dz \, d\theta \tag{2.27}$$

$$P(K) = \frac{1}{(\sigma^2/H) + 1)} \tag{2.28}$$

$$P_K(S) = \int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z \to \infty} \exp\left\{ -[\frac{2}{K} (1 - \varepsilon \cos \theta) + \frac{1}{\sigma^2}] z \right\} \frac{1}{2\pi\sigma^2} \, dz \, d\theta \tag{2.29}$$

$$= - \int_{\theta=0}^{\theta=2\pi} \int_{z=0}^{z \to \infty} \frac{1}{2\pi\sigma^2} \exp\left\{ -[\frac{2}{K} (1 - \varepsilon \cos \theta) + \frac{2}{H} + \frac{1}{\sigma^2}] z \right\} dz \, d\theta \tag{2.30}$$

Recognizing that the first double integral is exactly the same as before we can simplify to

$$P_K(S) = \frac{K}{2\sigma^2 \sqrt{\left(1 + \frac{K}{2\sigma^2}\right)^2 - \varepsilon^2}} - \int_{\theta=0}^{\theta=2\pi} \frac{1}{2\pi\sigma^2 \left(\frac{2}{K} - \frac{2\varepsilon}{K} \cos \theta + \frac{2}{H} + \frac{1}{\sigma^2} \right)} \, d\theta \tag{2.31}$$

$$P_K(S) = P(S) - \frac{K}{4\pi\sigma^2} \int_{\theta=0}^{\theta=2\pi} \frac{d\theta}{\left(1 + \frac{K}{2\sigma^2} + \frac{K}{H}\right) + (-\varepsilon) \cos \theta} \qquad (2.32)$$

The integration can be done by using the same formula as for the computation of $P_K(S)$, except that this time b is:

$$b = \left(1 + \frac{K}{2\sigma^2} + \frac{K}{H}\right) \qquad (2.33)$$

From this it follows:

$$P_K(S) = P(S) - \frac{K}{2\sigma^2 \sqrt{\left(1 + \frac{K}{2\sigma^2} + \frac{K}{H}\right)^2 - \varepsilon^2}} \qquad (2.34)$$

$$P_K(S) = \frac{K}{2\sigma^2} \left[\frac{1}{\sqrt{\left(1 + \frac{K}{2\sigma^2}\right)^2 - \varepsilon^2}} - \frac{1}{\sqrt{\left(1 + \frac{K}{2\sigma^2} + \frac{K}{H}\right)^2 - \varepsilon^2}} \right] \qquad (2.35)$$

After having derived the desired probabilities as summarized on the following figure, we are able to evaluate the different rates of temporary and permanent suppression.

By assuming a certain fire rate λ_f with which the artillery is firing in the area where the anti-tank gunners are located, the different rates will have the following form.

Two-event model (unsuppressed-suppressed)

Rate of suppression $\lambda_s = \lambda_f \; P(S)$ (2.36)

Four-event-model (unsuppressed-suppressed-survived-killed)

Rate of suppression $\lambda_s = \lambda_f \cdot P_k(S)$ (2.37)

Rate of killing $\lambda_k = \lambda_f \, P(K)$ (2.38)

One important rate must still be developed; it is the rate of rise from a suppressed state. We model this by stochastic process methods.

$$P(S) = \frac{K}{2\sigma^2 \sqrt{\left(1 + \frac{K}{2\sigma^2}\right)^2 - \varepsilon^2}} \qquad (2.19)$$

$$P(K) = \frac{1}{\frac{\sigma^2}{H} + 1} \qquad (2.28)$$

$$P_K(S) = P(S) - \frac{K}{2\sigma^2 \sqrt{\left(1 + \frac{K}{2\sigma^2} + \frac{K}{H}\right)^2 - \varepsilon^2}} \qquad (2.34)$$

Figure 6 - SUMMARY OF THE DERIVED FORMULAS

2. Rate Of Rise

The model so far represents a situation in which there are three possible outcomes. When any independent round impacts, the person in the foxhole is assumed to be either

suppressed,
not suppressed, or
is killed.

Naturally this is only true if we assume that he is always back up again when the next round impacts. This fact brings up the necessity of considering the process along the time axis.

Because of the complexity of a stochastic process which reflects the behavior situation of an individual exposed to arriving artillery rounds, it seems useful to start with a very simple process, in which the time of suppression by a particular round is considered to be fixed. In a next step, this constant response time is randomized over the number of people under consideration, i.e. each individual has his own random time which however is assumed constant for the process itself.

Finally, the response time of a single person may be considered to be a random variable coming from a certain distribution.

Both approaches are simple, and represent a first attempt to describe the actual situation in different ways. Certainly the suppression time can also be considered as a function of both above mentioned processes or of the miss distance r and the aspect angle θ, or of a combination of all. But these dependencies would rather complicate the mathematical derivations and lead beyond this study.

Both processes consider rounds that arrive in the

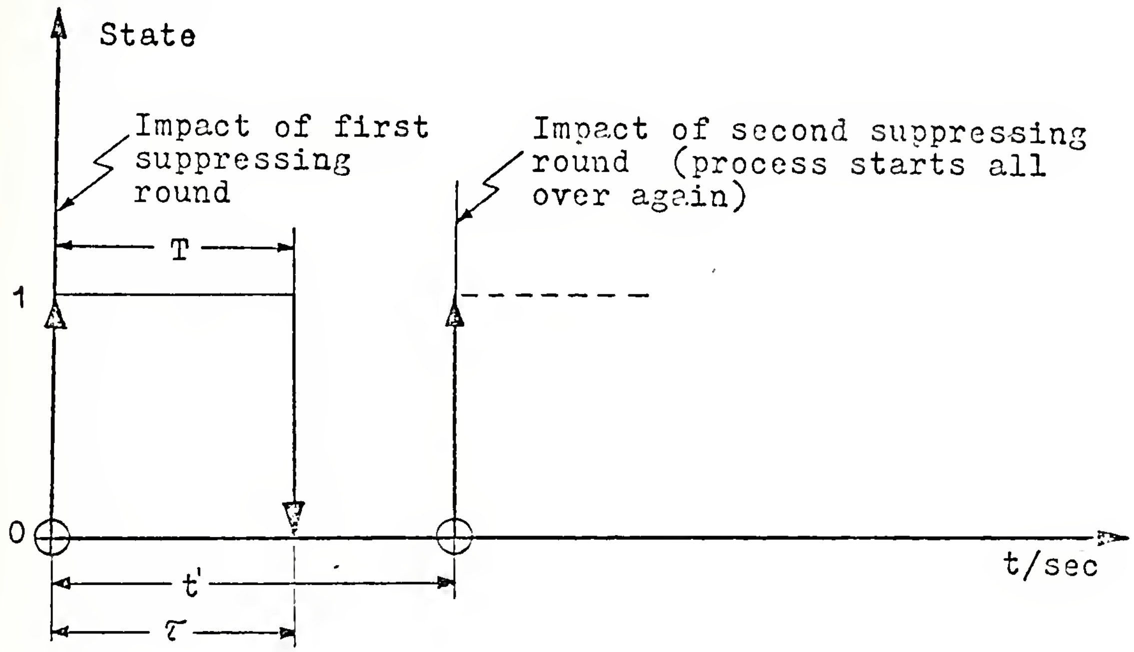
neighborhood of the foxhole in accordance with a Poisson Process having a rate λ_f. The model consists of two process states. One is the "up"-state which means that the person in the hole is unsuppressed and able to act according to his mission. The other state is termed the "down"-state which results from an incomming round and a possible reaction to it in the form of suppression. In this state the person is not able to fullfill his mission, he is physically down.

The first process assumes that the person in the foxhole changes from the "down"-state to the "up"-state only if he recognizes a gap of at least T time units before the appearance of the next round. This means that the "down"-state has a duration of exactly time T if no rounds arrive in the time interval (0,T). The time T in connection with such a process may be called the critical gap. [Gaver,p.481]. The following figure shows the basic relationship:

State 0 "up"
State 1 "down"

T = critical gap

FIRST CASE : $t' > T$



SECOND CASE : $t' \leqslant T$

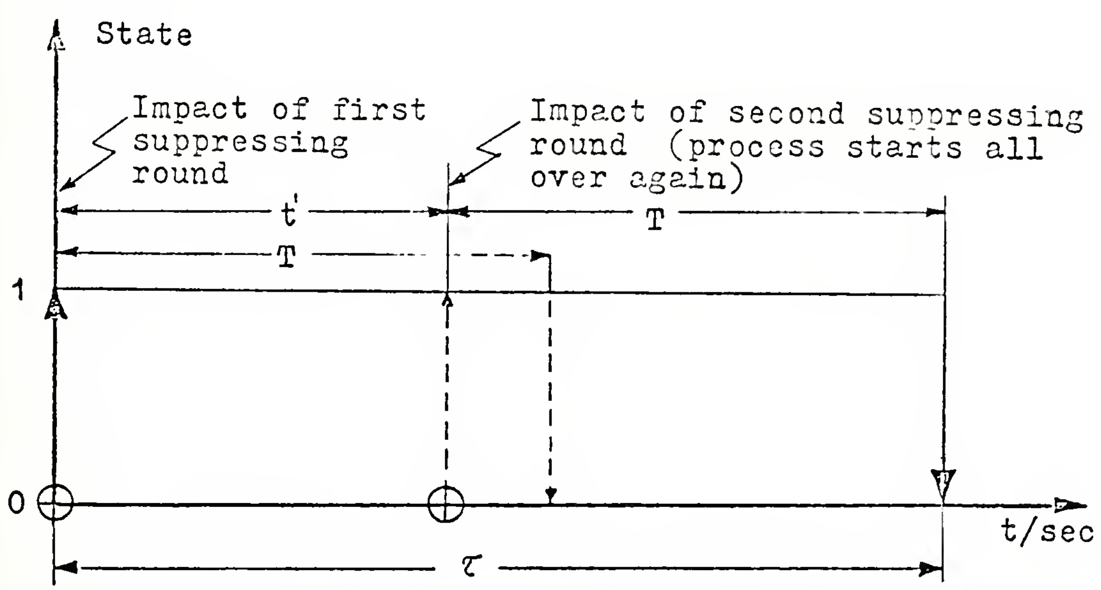


Figure 7 - SCHEMATIC DESCRIPTION OF THE MODEL

For the time being T is assumed fixed. However it is more reasonable to describe it by a random variable, since the time gap T is determined by factors like learning, accustoming to, or overcomming of, fear, stress etc. The assumption of a fixed T has to be changed in a later step, where it will be defined as a random variable.

Furthermore, the change from the "up"-state to the "down"-state is accomplished by arriving rounds to which the person reacts through suppression ("down"-state) with the earlier computed probability of suppression P(S).

Now let τ be equal to the total time of being continually suppressed. It is possible to define τ in the following way:

$$\tau = \begin{cases} T \text{ if no suppressing round arrives in } (0,T) \\ t'+\tau' \text{ if the next suppressing round falls} \\ \text{before } T. \end{cases} \tag{2.39}$$

In this case the random variable τ' is an independent version of the random variable τ.

According to this definition it is possible to compute the expected value of τ which represents the mean time spent in the "down"-state, or the mean time of being continually suppressed. The symbol used for it will be $E[\tau]$.

In order to get the arrival rate of suppressing rounds which contribute to the time of being suppressed, we have to multiply the fire rate λ_f with the probability of suppression, P(S), which was evaluated in chapter II.B.1. We obtain

$$\lambda_s = \lambda_f \cdot P(S) \tag{2.36}$$

First we have to state the following two expectations:

$$E[\tau | \text{no round in } T] = T$$
$$E[\tau | \text{round in } T] = t' \tag{2.40}$$

where t' = time of second suppressing round. Removing the condition on t' leads to expected time of being suppressed under the influence of incomming rounds.

$$E[\tau] = T\cdot e^{-\lambda_s T} + \int_0^T e^{-\lambda_s t'} \lambda_s dt'[t' + E[\tau]] + E[\tau]\,[1 - e^{-\lambda_s T}] \qquad (2.41)$$

$$E[\tau] = T\cdot e^{-\lambda_s T} + \lambda \left[\frac{e^{-\lambda_s t}}{\lambda_s^2} (-t-1) \right]_0^T + E[\tau]\,(1 - e^{-\lambda_s T})$$

$$E[\tau] = T\, e^{-\lambda_s T} + \frac{1}{\lambda}\,[1 - (1 + \lambda_s T)\, e^{-\lambda_s T}] + E[\tau](1 - e^{-\lambda_s T})$$

$$E[\tau] = T + \frac{1}{\lambda e^{-\lambda_s T}}\,[1 - (1 + \lambda_s T) e^{-\lambda_s T}]$$

$$E[\tau] = \frac{1}{\lambda_s}\,(e^{\lambda_s T} - 1) \qquad (2.42)$$

inserting $\lambda_s = \lambda_f \cdot P(S)$ we obtain

$$E[\tau] = \frac{1}{\lambda_f P(S)}\,(e^{\lambda_f P(S)\cdot T} - 1) \qquad (2.43)$$

Extending the idea for a fixed time gap by applying it to a group of people separately, we are able to reformulate the process in the following way:

Assume a group of persons in which each individual sticks to a certain but random time gap T when responding to a suppressive round, and assume that this random variable T based on the group is distributed with a density $f_p(t)$:

$$T \sim f_p(t) \qquad (2.44)$$

The subscript P indicates the origin of the density (people). We may express τ which is defined as before as follows:

$$\tau = \begin{cases} t \text{ if no suppressing round arrives in } (0,t) \text{ or } t'>t \\ \text{where } t \text{ is the time of the first suppressing round.} \\ t'+\tau' \text{ if } t'\leq t \end{cases} \qquad (2.45)$$

Remark, τ' has the same distribution as τ. A similar derivation as before leads to the following result.

$$E[\tau|T=t] = t\int_t^\infty \lambda_s e^{-\lambda_s t'} dt' + \int_0^t [t'+E[\tau|T=t]]\lambda_s e^{-\lambda_s t'} dt' \qquad (2.46)$$

Recall, the expression $\lambda_s e^{-\lambda_s t}$ is the density of incoming suppressing rounds.

$$E[\tau|T=t] = te^{-\lambda_s t} + \int_0^t t'\lambda_s e^{-\lambda_s t'} dt' + E[\tau|T=t](1 - e^{-\lambda_s t})$$

$$E[\tau|T=t] = \frac{1}{\lambda_s} - \frac{1}{\lambda_s} e^{-\lambda_s t} + E[\tau|T=t](1 - e^{-\lambda_s t}) \qquad (2.47)$$

Solving for $E[\tau|T=t]$, we find:

$$E[\tau|T=t] = \frac{1}{\lambda_s}(e^{\lambda_s t} - 1) \qquad (2.48)$$

or replacing λ_s again by $\lambda_s = \lambda_f \cdot P(s)$

$$E[\tau|T=t] = \frac{1}{\lambda_f P(S)} (e^{\lambda_f P(S) t} - 1) \qquad (2.49)$$

So far this value represents the expected duration spent in the "down state" given a particular time gap t for a certain individual. Clearly the conditional expectation has the same form as the unconditional expectation $E[\tau]$ (see formula (2.43)) for a fixed time gap T.

Removing the condition in equation (2.49) we find the expected duration time of suppression based on the considered population.

$$E[\tau] = \int_0^\infty E[\tau|T=t] \cdot f_p(t) dt \qquad (2.50)$$

$$E[\tau] = \int_0^\infty \frac{1}{\lambda_f P(S)} (e^{\lambda_f P(S) t} - 1) \cdot f_p(t) dt \qquad (2.51)$$

$$E[\tau] = \frac{1}{\lambda_f P(S)} \int_0^\infty e^{\lambda_f P(S) t} f_p(t) dt - \frac{1}{\lambda_f P(S)} \qquad (2.52)$$

For an evaluation of this expectation, the density $f_p(t)$ has to be known.

A further variation of this process leads to the second approach. Here the time gap T for a certain individual varies randomly according to a particular distribution. This approach is limited to one individual, and will not be extended to a group.

Suppose an individual reacts with a random time gap t_i

$$T=t_i \qquad i=1,2,3,\ldots$$

on any incoming suppressing round i, and assume that this random variable T is distributed with

$$T \sim f_T(t) \tag{2.53}$$

The subscript T indicates the origin of the density (time). The duration τ is defined as in formula (2.45) before. The conditional expectation can be written as:

$$E[\tau|T_1=t] = te^{-\lambda_s t} + \int_0^t (t' + E[\tau])\,\lambda_s e^{-\lambda_s t'}\,dt' \tag{2.54}$$

This time τ is conditioned on the individual's first chosen time gap T.

$$E[\tau|T_1=t] = \frac{1}{\lambda_s}(1 - e^{-\lambda_s t}) + E[\tau](1- e^{-\lambda_s t}) \tag{2.55}$$

Removing the condition in equation (2.55) we can express the expected duration time of suppression based on the individual's time gap distribution.

$$E[\tau] = \int_0^\infty E[\tau|T_1=t]\cdot f_T(t)dt \tag{2.56}$$

$$E[\tau] = \int_0^\infty \left[\frac{1}{\lambda_s}(1 - e^{-\lambda_s t}) + E[\tau](1 - e^{-\lambda_s t}) \right] f_T(t)dt \tag{2.57}$$

Solving for the expected value E[τ] and inserting

$$\lambda_s = \lambda_f \cdot P(S) \tag{2.36}$$

we find:

$$E[\tau] = \frac{\int_0^\infty (1 - e^{-\lambda_f P(S)t})\, f_T(t)dt}{\lambda_f P(S)\left[1 - \int_0^\infty (1 - e^{-\lambda_f P(S)t})\, f_T(t)dt\right]} \tag{2.58}$$

or expressing in Laplace transform with s as an argument:

$$E[\tau] = \frac{1}{\lambda_f P(S)} \left[\frac{1}{\hat{f}(s)} - 1\right] \tag{2.59}$$

A further evaluation of this value requires the distribution $f_T(t)$ of T.

comparing both expectations,

Expected duration time of suppression based on the population

$$E[\tau] = \frac{1}{\lambda_f P(S)} \left[\int_0^\infty e^{\lambda_f P(S)t}\, f_P(t)dt - 1\right] \tag{2.52}$$

and expected duration time of suppression based on the individual's time gap distribution

$$E[\tau] = \frac{1}{\lambda_f P(S)} \left[\frac{1}{\hat{f}(s)} - 1\right] \tag{2.59}$$

We observe that they are different. However if we suppose that both distributions $f_p(t)$ and $f_T(t)$ are concentrated at T=z (delta function) it is possible to reduce both expressions to the very first derivation (2.48) where the the time gap T was fixed.

The reciprocals of these expectations approximate the rate at which the foxhole occupant (e. g. member of a group of antitank gunners returns from the suppressed state into the unsuppressed state) returns back to continue his mission.

$$\lambda_u = \frac{1}{E[\tau]} \qquad (2.60)$$

where E[τ] represents any of the derived expectations.

In summary the three rate coefficients develcped are:

$$\lambda_k = \lambda_f \cdot P(K) \qquad (2.38)$$

$$\lambda_s = \lambda_f \cdot P_k'(S) \qquad (2.37)$$

$$\lambda_u = \frac{1}{E[\tau]} \qquad (2.60)$$

If killing as an additional event is considered, the rate of rise is computed with the same formula (2.60) except that this time $P_k(S)$ is used instead of P(S).

C. SUPPRESSION MODEL

The model which will be developed in this section can be set schematically in the following framework:

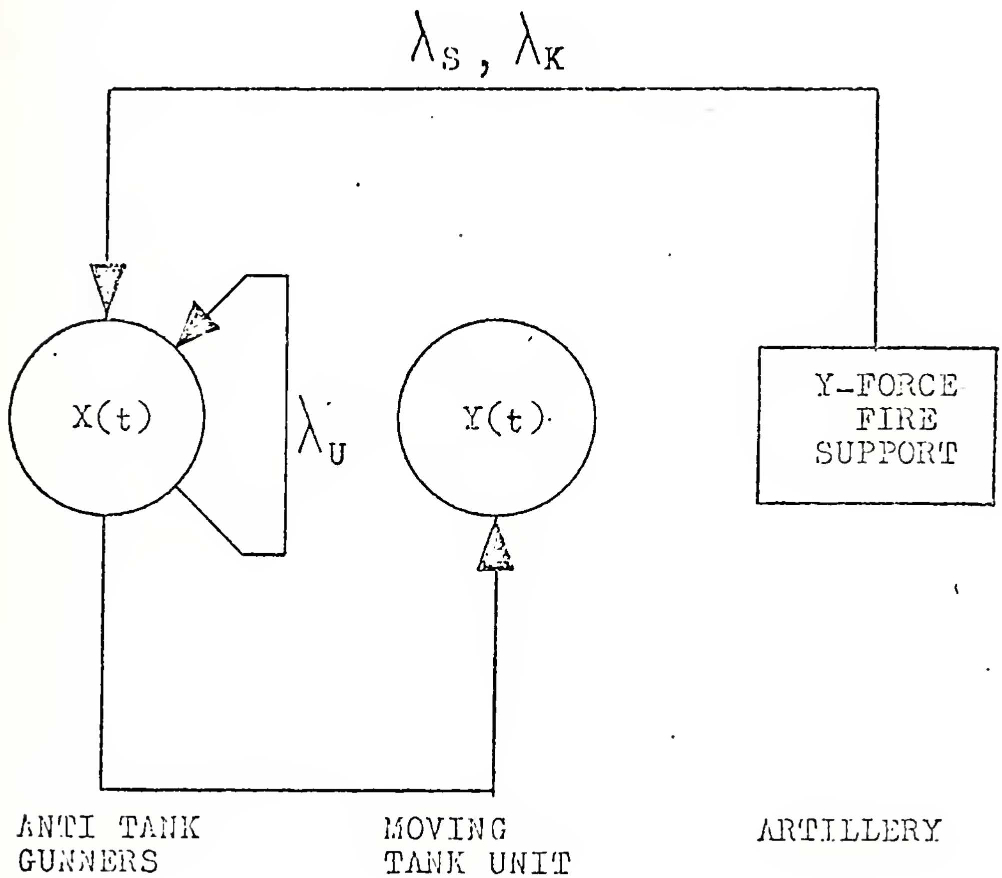


Figure 8 - SCHEMATIC REPRESENTATION OF THE SITUATION TO BE MODELED

This analysis is restricted to the effects on the anti-tank gunners... in this case the X-forces. It is behavior of members of the X-force that was modeled previcisly.

The approach to formulate the situation of the anti-tank gunners is done by using the ideas of Lanchester-type equations and their further development [Tayler,1978,p.20].

The differential equations representing the model are all deterministic in the sense that each of them will always yield the same output for a given set of input data. Even though combat between military forces is a complex random process. These equations shed light on combat dynamics and may be useful in defense planning studies.

The basic idea is that artillery forces use "area"-fire to suppress or eliminate forces like anti-tank gunners. "Area" in this context means the fact that

> the artillery unit "knows" the area in which to shoot, but does not know the location of each anti-tank gunner,
>
> the anti-tank gunners are "invisible" to the artillery unit.

If we further assume homogeneous forces of X, it is possible to set up differential equations which model the rate of change of the X-forces:

$$\frac{dX_a(t)}{dt} = -\lambda_s X_a(t) - \lambda_k X_a(t) + \lambda_u X_s(t) \qquad (2.61)$$

$$\frac{dX_s(t)}{dt} = \lambda_s X_a(t) - \lambda_u X_s(t) \qquad (2.62)$$

Remark: Clearly λ_s here is taken from the 4-event-model. The

variables $X_a(t)$ and $X_s(t)$ are respectively the number of the X-forces which are either active in the foxhole (able to use guns and anti-tank-weapons), $X_a(t)$, or suppressed, $X_s(t)$.

In this simplified structure, the two equations describe the most essential factors of the assumed situation. They are mathematically approximate, because the solution of the two differential equations will furnish the number of gunners active on the battlefield and the number of gunners supppressed at any point in time. The derivation uses the Laplace transformation:

$$\mathcal{L}\left\{\frac{dX(t)}{dt}\right\} = sx(s) - X(0) \qquad (2.63)$$

$$\mathcal{L}\{X(t)\} = x(s)$$

We perform Laplace transformation upon (2.61) and (2.62)

$$sx_a(s) - X_a(0) = -(\lambda_s + \lambda_k)\, x_a(s) + \lambda_u x_s(s) \qquad (2.64)$$

$$sx_s(s) - X_s(0) = \lambda_s x_a(s) - \lambda_u x_s(s) \qquad (2.65)$$

Solving these two equations for $X_a(t)$ and $X_s(t)$ and translating them back to $X_a(t)$ and $X_s(t)$ gives the desired time dependent quantities:

$$-X_a(0) = -(\lambda_s + \lambda_k + s)\, x_a(s) + \lambda_u x_s(s) \qquad (2.66)$$

$$-X_s(0) = \lambda_s x_a(s) - (\lambda_u + s)\, x_s(s) \qquad (2.67)$$

$$-X_a(0)[\lambda_u + s] = -(\lambda_s + \lambda_K + s)(\lambda\ + s)\, x_a(s) + \lambda_u(\lambda_u + s)\, x_s(s)$$

$$-X_s(0)\,\lambda_u = \lambda_s\lambda_u x_a(s) - (\lambda_u + s)\lambda_u\, x_s(s)$$

$$-X_a(0)(\lambda_u + s) - X_s(0)\lambda_u = \lambda_s\lambda_u x_a(s) - (\lambda_s + \lambda_k + s)(\lambda_u + s)\, x_a(s)$$

$$x_a(s) = \frac{X_a(0)\,\lambda_u + X_s(0)\,\lambda_u + sX_a(0)}{(\lambda_s + \lambda_k + s)(\lambda_u + s) - \lambda_s\lambda_u} \qquad (2.68)$$

multiplying (2.66) and (2.67) with λ_s and $(\lambda_s + \lambda_k + s)$ respectively we receive:

$$x_s(s) = \frac{X_a(0)\lambda_s + X_s(0)(\lambda_s + \lambda_k) + sX_s(0)}{(\lambda_s + \lambda_k + s)(\lambda_u + s) - \lambda_s\lambda_u} \tag{2.69}$$

Suppose that at t=0 the number of active gunners (able to watch and shoot) is equal to X_0 and the number of gunners being suppressed (down in the foxhole) is equal to 0,

for i.e. t=0

$$X_a(0) = X_0$$
$$X_s(0) = 0 \tag{2.70}$$

the equations for $X_a(s)$ and $X_s(s)$ can be rewritten:

$$x_a(s) = \frac{X_0\lambda_u}{s^2 + s(\lambda_s + \lambda_k + \lambda_u) + \lambda_k\lambda_u} + \frac{X_0 s}{s^2 + s(\lambda_s + \lambda_k + \lambda_u) + \lambda_k\lambda_u} \tag{2.71}$$

$$x_s(s) = \frac{X_0\lambda_s}{s^2 + s(\lambda_s + \lambda_k + \lambda_u) + \lambda_k\lambda_u} \tag{2.72}$$

Using the Laplace correspondence:

$$\mathcal{L}\left\{\frac{e^{at} - e^{bt}}{a - b}\right\} = \frac{1}{s^2 + (-a-b)s + ab} \quad \text{for } a \neq b$$

$$\mathcal{L}\left\{\frac{ae^{at} - be^{bt}}{a - b}\right\} = \frac{s}{s^2 + (-a-b)s + ab} \quad \text{for } a \neq b \tag{2.73}$$

where $a \cdot b = \lambda_k \lambda_u$.

and $-a - b = \lambda_s + \lambda_k + \lambda_u$ (2.74)

hence $$a = \frac{1}{2}[-(\lambda_s + \lambda_k + \lambda_u)] + \sqrt{(\lambda_s + \lambda_k + \lambda_u)^2 - 4\lambda_k\lambda_u}$$

and $$b = \frac{1}{2}[-(\lambda_s + \lambda_k + \lambda_u)] - \sqrt{(\lambda_s + \lambda_k + \lambda_u)^2 - 4\lambda_k\lambda_u}$$ (2.75)

The equation for $X_a(t)$ and $X_s(t)$ then are:

$$X_a(t) = \frac{X_0}{a - b}\ [(\lambda_u + a)e^{at} - (\lambda_u + b)e^{bt}] \quad (2.76)$$

$$X_s(t) = \frac{X_0}{a - b}\ \lambda_s[e^{at} - e^{bt}] \quad (2.77)$$

Since λ_s, λ_k and λ_u have to be always greater or equal to zero

$$\lambda_s \geq 0$$

$$\lambda_k \geq 0 \quad (2.78)$$

$$\lambda_u \geq 0$$

The constants a and b are always negative and real numbers.

$$a \leq 0$$

$$b \leq 0 \quad (2.79)$$

This leads to the basic shape of the function

$$X_a(t) = f(t)$$

$$X_s(t) = f(t) \quad (2.80)$$

shown in the following figure.

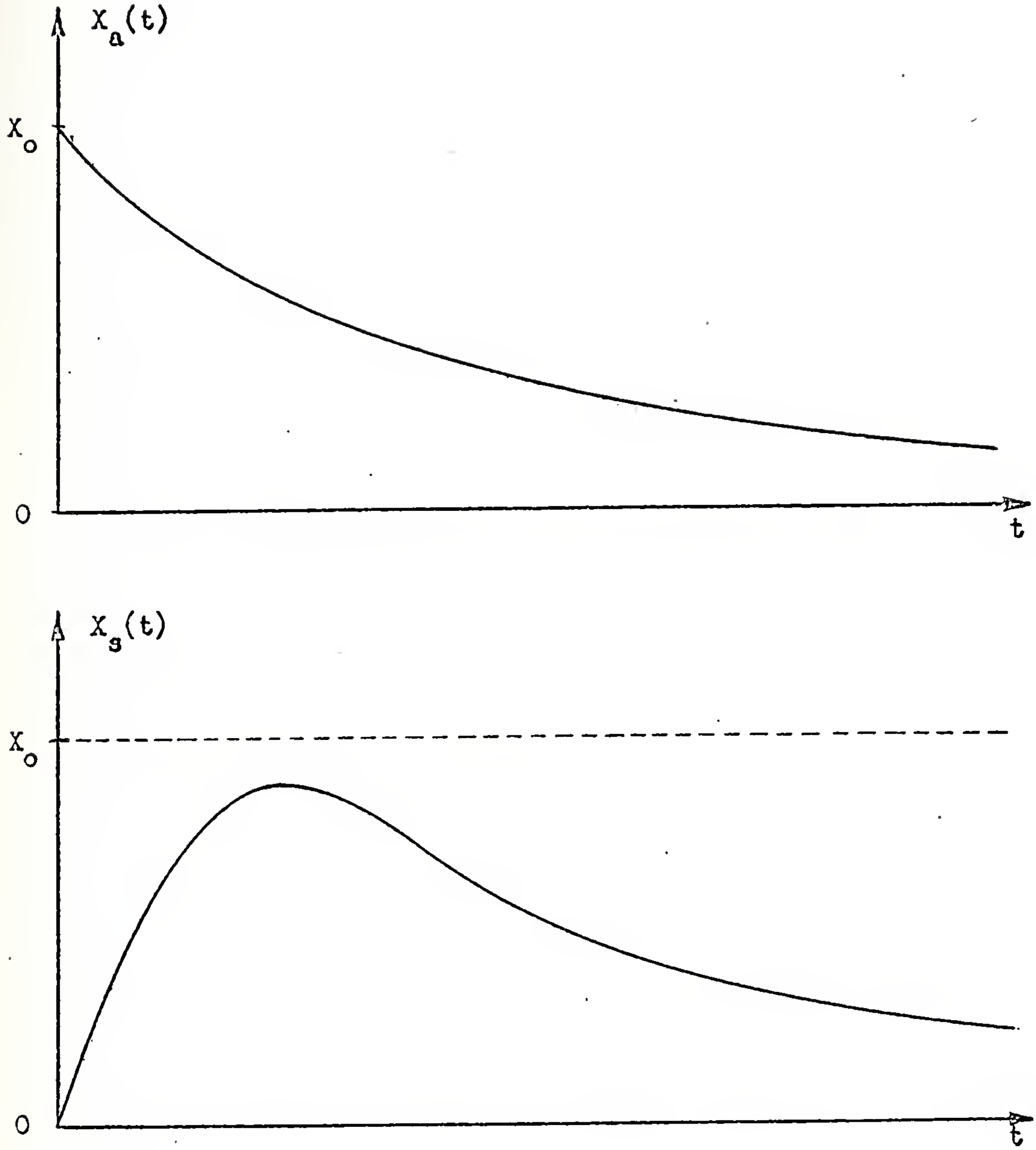


Figure 9 - FUNCTIONS $X_a(t)$ AND $X_s(t)$

In discussing the functions $X_a(t)$ and $X_s(t)$ the following properties can be seen.

for $t = 0$

$$X_a(0) = \frac{X_0}{a - b} [(\lambda_u + a)e^{a\cdot 0} - (\lambda_u + b)e^{b\cdot 0}] = X_0 \qquad (2.81)$$

$$X_s(0) = \frac{X_0}{a - b} \lambda_s [e^{a\cdot 0} - e^{b\cdot 0}] = 0 \qquad (2.82)$$

for $t = \infty$

$$X_a(\infty) = \lim_{t \to \infty} \frac{X_0}{a - b} [(\lambda_u + a)\cdot e^{at} - (\lambda_u + b)\cdot e^{bt}] = 0 \qquad (2.83)$$

$$X_s(\infty) = \lim_{t \to \infty} \frac{X_0}{a - b} \lambda_s [e^{at} - e^{bt}] = 0 \qquad (2.84)$$

for $t = t_{max}$ (i.e. max values for $X_a(t)$ and $X_s(t)$)

$X_a(t)$ has no max in $(0,\infty)$, i.e. max value is at $t = 0$

$$X_a(t) = X_0 \qquad (2.85)$$

$X_s(t)$ has a max at

$$t_{max} = \frac{\ln(a/b)}{b - a} \quad \text{with} \qquad (2.86)$$

$$X_s(t_{max}) = \frac{X_0}{a - b} \lambda_s [e^{at_{max}} - e^{bt_{max}}] < X_0 \qquad (2.87)$$

The foregoing model assumed a rate of killing in addition to a suppression rate and a rising rate. It is possible to simplify this model by leaving out the third rate. It will be shown in chapter III.C., that this rate

$$\lambda_k = \lambda_f \cdot P(K) \qquad (2.38)$$

for such a scenario may be very small in comparison to the other two rates.

For this reason we can state that leaving out the killing rate will not drastically oversimplify the model, yet it will simplify the computational procedure in obtaining the wanted dependency between time and the number of gunners active or suppressed on the field.

This leads to the following set-up of differential equations:

$$\frac{dX_a(t)}{dt} = -\lambda_s X_a(t) + \lambda_u X_s(t) \tag{2.88}$$

$$\frac{dX_s(t)}{dt} = \lambda_s X_a(t) - \lambda_u X_s(t) \tag{2.89}$$

where the total sum of people either suppressed or active on the battle field is equal to a constant X_o (no killing)

i.e. $X_o = X_a(t) + X_s(t)$

which enables us to rewrite the equations:

$$\frac{dX_a(t)}{dt} = -\lambda_s X_a(t) + \lambda_u (X_0 - X_a(t)) \tag{2.90}$$

$$\frac{dX_s(t)}{dt} = \lambda_s (X_0 - X_s(t)) - \lambda_u X_s(t) \tag{2.91}$$

Using again Laplace transformation as earlier:

$$sX_a(s) - X_0(0) = -\lambda_s X_a(s) + \frac{\lambda_a X_a}{s} - \lambda_a X_a(s) \tag{2.92}$$

$$sX_s(s) - X_s(0) = \frac{\lambda_s X_0}{s} - \lambda_s X_s(s) - \lambda_u X_s(s) \tag{2.93}$$

$$x_a(s) = \frac{\lambda_u X_0}{s[s + (\lambda_s + \lambda_u)]} + \frac{X_a(0)}{s + (\lambda_s + \lambda_u)}$$

$$x_s(s) = \frac{\lambda_s X_0}{s[s + (\lambda_s + \lambda_u)]} + \frac{X_a(0)}{s + (\lambda_s + \lambda_u)}$$

using the Laplace transform:

$$\mathcal{L}\left\{\frac{1}{a}(1-e^{-at})\right\} = \frac{1}{s(s + a)}$$

$$\mathcal{L}\{e^{-at}\} = \frac{1}{s + a} \tag{2.94}$$

for both equations we find $X_a(t)$ and $X_s(t)$ for the simplified model.

The rate λ_s has to be derived this time from the probability of suppression P(S) without killing included as a possible event. We recall;

$$P(S) = \frac{K}{2\pi\sigma^2 \sqrt{\left(1 + \frac{K}{2\sigma^2}\right)^2 - \varepsilon^2}}, \quad (2.19) \qquad \lambda_s = \lambda_f \cdot P(S) \quad (2.36)$$

Hence

$$X_a(t) = \lambda_u X_0 \frac{1}{\lambda_s + \lambda_u} (1 - \exp[-(\lambda_s + \lambda_u)t]) + X_a(0) \exp[-(\lambda_s + \lambda_u)t] \tag{2.95}$$

$$X_s(t) = \lambda_s X_0 \frac{1}{\lambda_s + \lambda_u} (1 - \exp[-(\lambda_s + \lambda_u)t]) + X_s(0) \exp[-(\lambda_s + \lambda_u)t] \tag{2.96}$$

for t = 0

$$X_a(0) = X_0, \qquad X_s(0) = 0 \tag{2.97}$$

The equation for $X_a(t)$ and $X_s(t)$ then are:

$$X_a(t) = \frac{X_0}{\lambda_s + \lambda_u} [\lambda_u + \lambda_s \cdot \exp[-(\lambda_s + \lambda_u)t]] \tag{2.98}$$

$$X_s(t) = X_0 \frac{\lambda_s}{\lambda_s + \lambda_u} (1 - \exp[-(\lambda_s + \lambda_u)t]) \tag{2.99}$$

Remark: For $t \to \infty$ the sum of both assymptotic values of $X_a(t)$ and $X_s(t)$ is equal to X_0.

The basic shape of these two functions $X_a(t)$ and $X_s(t)$ can be seen in the following Fig 10.

If we compare this result to the result on figure 9 we can see that the addition of killing to the model has an influence on the shape of the functions. So is e. g. the value for $X_a(t)$ and $X_s(t)$ in the first model (figure 9) for large times approximately zero, while in this case here (figure 10), the total number of people on the field ($X_a(t)$ + $X_s(t)$) is always constant, i. e. both functions approaches to a limit value not equal to zero.

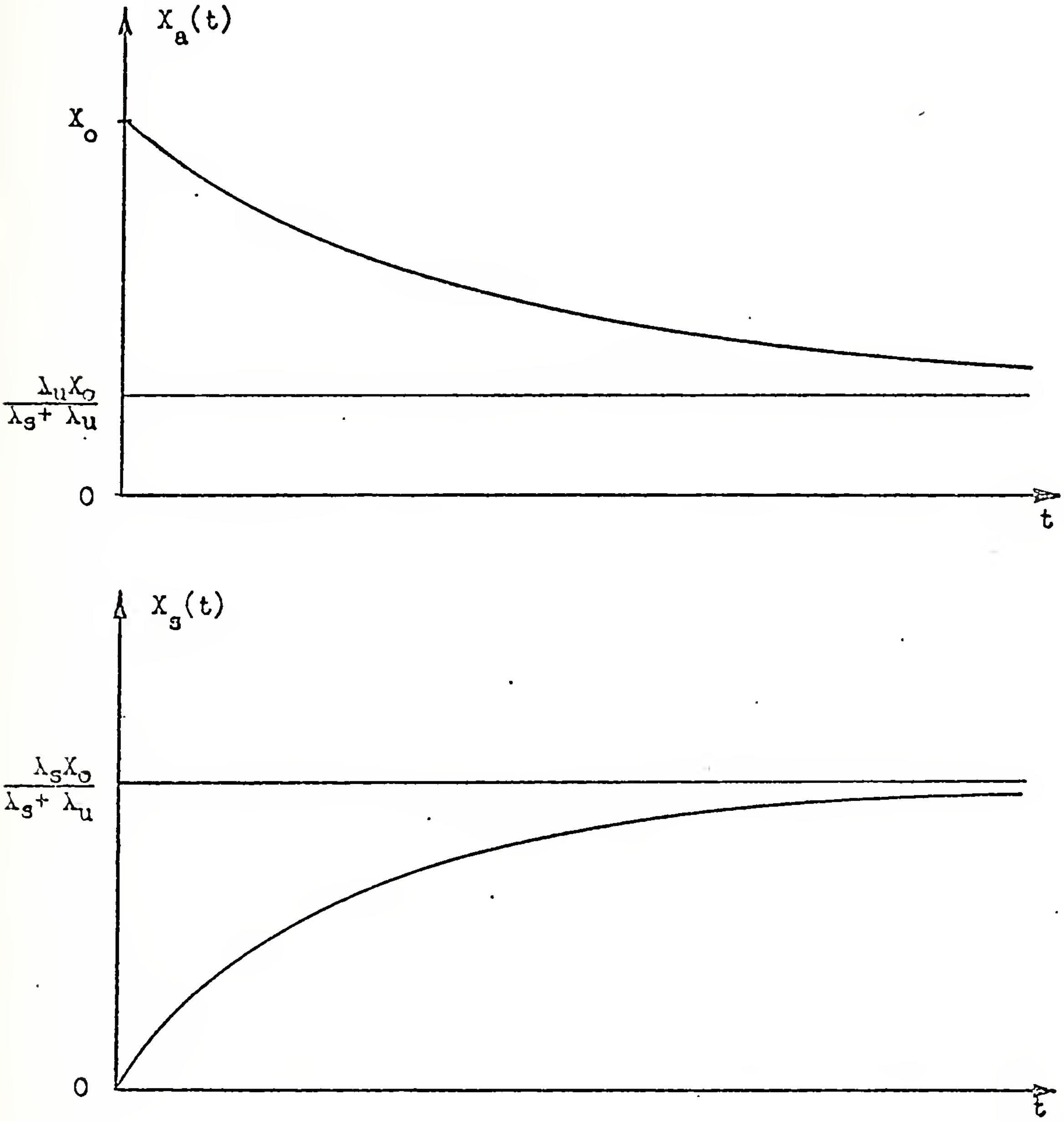


Figure 10 - FUNCTIONS $X_a(t)$ AND $X_s(t)$

III. FITTING EXPERIMENTAL DATA DESCRIBING SUPPRESSION

A. EXPERIMENTAL DESIGN

1. General Aspects

The mathematical model developed in the chapter before shall now be supported by an experiment which was conducted by the US Army Combat Developments Experimentation Command (CDEC), Fort Ord, California.

The experiment described here belongs to a series of similar experiments, all dealing with the objective of collecting and analyzing data of the suppression process. The data of this particular part of the CDEC experiment were based on the relationship between suppression and distance. This analysis tries to make use of the data by analyzing the relationship among suppression, distance, and angle. In a further experiment conducted by CDEC, the objective was also to include the angle as an additional variable for the suppression effect.

2. Setup And Realisation

The data were taken from a part of the experiment which was executed at Fort Hunter Liggett, California.

Four foxhole-bunkers were constructed which guaranteed the safety of the players as well as reproduced the real scenario as close as possible. Their tops were below ground level, and covered with several layers of wire mesh and steel plates. This provided overhead protection from fragmentation. Each bunker was equipped with a mirror system which allowed the player to view events in front of

his position.

A pop-up silhouette was installed forward of each player position. The player was able to control the silhouette as well as the cover of the mirror system, i.e. when the cover was opened (allowing the player to view down-range), the silhouette was in exposed posture. He was asked to respond as if he would be in the position of the pop-up silhouette.

Each bunker was connected to controll bunkers by communication and instrumentation wires and power bunkers for data recording and supply. In the forefield of the bunkers, different types of projectiles or equivalent charges (81 mm,105 mm,and 155 mm among others) were placed and randomly detonated one at a time with the time between detonations randomly selected from three possibilities of ten, fifteen, and twenty seconds. The figure on the next page shows the schematic setup of the rounds, the location of the bunkers and the angle of sight for each foxhole. It can be seen that the explosions were visible to some players but not to others. Since each type of ammunition has a different lethal radius, it was necessary to have different range configurations for each type. The aspect angle and the miss distances from the foxholes to the different points of detonation are summarized in appendix A for each ammunition separately. Since all of the projectiles used in this part of the experiment were statically detonated, it was not possible to model the kinetic contribution to the terminal effects. In order to keep the fragmentation pattern as close as possible to those of incoming rounds, the projectiles were supported with sandbags [Suppex,p.A-12].

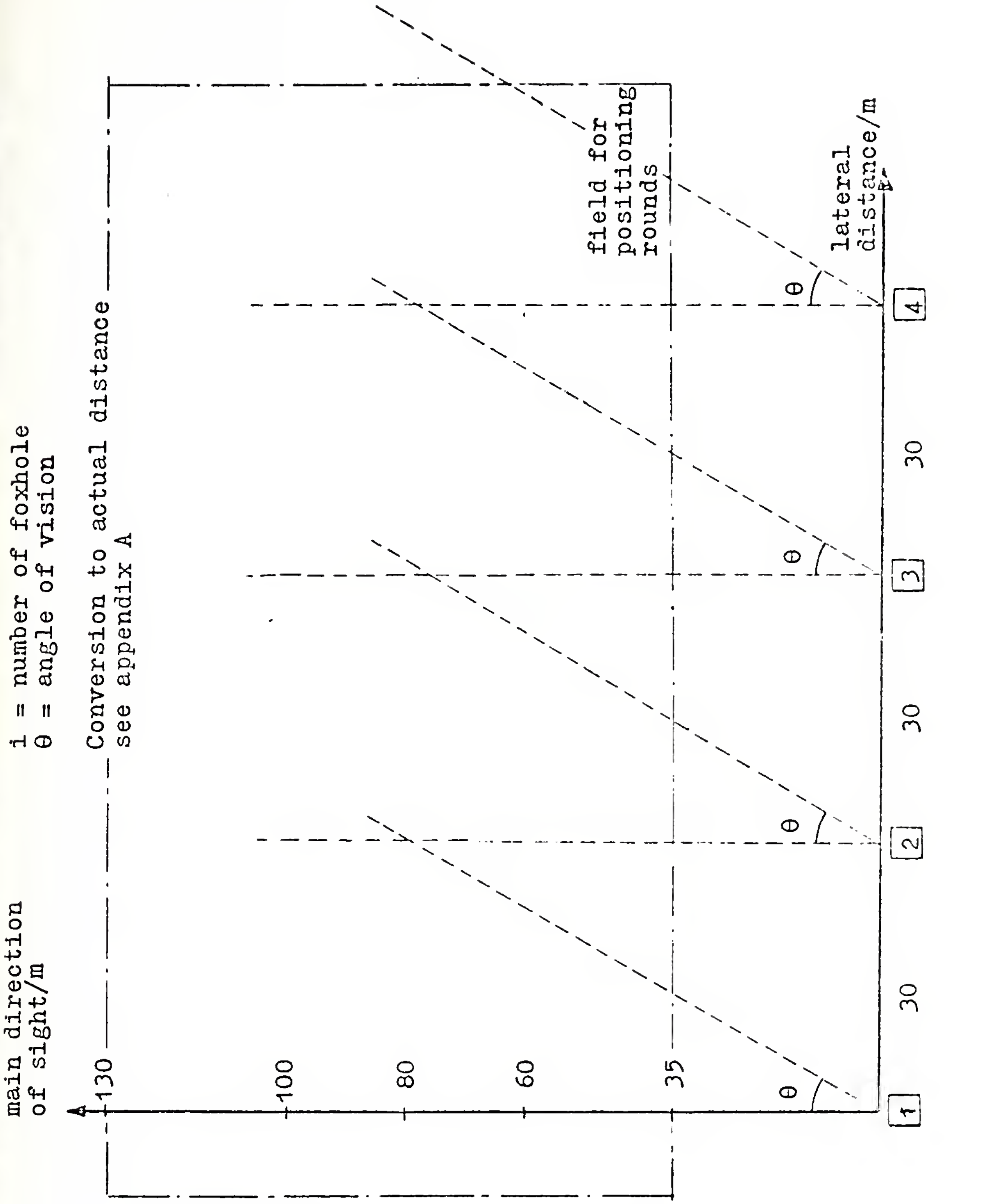


Figure 11 – SCHEMATIC RANGE FOR 81/105/155 mm SUPPRESSION EXPERIMENT (SUPPEX II) CDEC

The players were divided into two four-man teams. For each trial, members of one team occupied individual foxholes and provided the performance data for that trial. The mission of the players was to track moving target tanks by operating the periscopes. This mission was interrupted by the players responses to detonations in case he was suppressed (change of his state to "down"). The state change and the pericd of suppression were automatically recorded.

3. Presentation Of The Data

Appendix A summarizes the data which are the basis for the succeeding data analysis and serve tc evaluate the parameters and rates for the model.

E. DATA ANALYSIS

1. Parameters Of The Model

The data presented in the section above are separated into three different sets:

Set cne consists of data related to the conditional frequency of suppression $\hat{P}(S/\theta,r)$ and the time of suppression when rounds of caliber 81 mm were fired. Specifically, if n trials were made under conditions (r,θ), the number s_i cf suppression was tabulated. Then s_i/n is an estimate of $P(S/r_i,\theta_i)$.

Sets two and three provide the same data except that they are related to 105 mmm rcunds and 155 mm rcunds.

The main part of this analysis is to make use of the data in crder to estimate the parameters of the model described in

chapter II, and to see how well the real world situation can be described by the model developed earlier.

The model is fitted (numerical values of the parameters are determined) by the method of least squares. [De Groot, p.527-538]. With the basic model being of the form:

$$Y_i = \beta_1 u_i + \beta_2 v_i + Err_i \qquad (3.1)$$

And the data are analyzed on the following two different assumptions; the second is certainly the most realistic:

Homoscedasticity of the data, i. e. the error variance of Err is assumed to be constant.

Heteroscedasticity of the data, i. e. we will assume that each error term Err is distributed with variance σ^2 ,where the variance is not constant over observations. Errors are also assumed to be independent. If convenient, error terms will be assumed to be approximately normally distributed.

These assumptions imply different regression methods. For the first assumption, the result of unweighted single step and iterative regression methods will be presented. On the assumption of heteroscedasticity, two special iterative methods will be used. When we allow for heteroscedasticity, ordinary least-squares estimation places the same weight on the observations which have small error variances as on those with large error variances. By applying a weighting regression, it is possible to adjust for the heteroscedasticity. So the two announced methods for the second assumption are iteratively weighted least squares regression methods.

In order to prepare the formula of the model for the regression, we apply a log-transformation to the equation (2.1) in chapter II B.1. This is a convenience, for it transforms the problem to one of linear fitting. We obtain the following result:

$$P(S/\theta,r) = \exp[-\frac{1}{K} r^2 (1 - \varepsilon \cos \theta)] \qquad (2.1)$$

$$\ell n\, P(S/\theta,r) = -\frac{1}{K} r^2 + \frac{\varepsilon}{K} r^2 \cos \theta \qquad (3.2)$$

Using the notation

$$y = \ln P(S/\theta,r)$$

$$u = r^2 \qquad (3.3)$$

$$v = r^2 \cdot \cos\theta$$

the equation can be applied to each datapoint i and the equation can be rewritten as:

$$y_i = -\frac{1}{K} \mu_i + \frac{\varepsilon}{K} v_i \qquad (3.4)$$

The model can then be expressed as follows:

$$Y_i = \beta_1 u_i + \beta_2 v_i + Err_i \qquad (3.1)$$

where the unknown parameters are: $\beta_1 = -\frac{1}{K}$ $\qquad \beta_2 = \frac{\varepsilon}{K}$ (3.5)

and the random variable

$$Y_i = \ln \hat{P}(S/\theta_i, r_i) \tag{3.6}$$

The "hat" on the letter P represents an estimate of the probability of suppression at (r,θ). The fitted regression plane for this function has the form:

$$\hat{y} = \hat{\beta}_1 u + \hat{\beta}_2 v \tag{3.7}$$

The log-transformation of the conditional frequency of suppression $\hat{P}(S/\theta,r)$ causes difficulties because of experimental results, which lead to an observed frequency of zero suppression. This fact influences the estimate $\hat{P}(S/\theta,r)$, that is used.

Having specified the model it is possible to apply the following different regression methods on the assumption of homoscedasticity:

In other words, suppose n_i observations were made at experimental conditions (r_i,θ_i) and of these s_i were successes. Then first consider the estimate

$$Y_i = \ln \hat{P}(S/r_i,\theta_i) = \ln(s_i/n_i) \tag{3.8}$$

where s_i is the number of suppressions, and n_i is the number of exposures to suppression, under condition i.

Consequently, the transformed response to conditions (r_i,θ_i) may be $\ln(\theta/n_i) = -\infty$ formally, causing embarassment. We will subsequently suggest alternative ways of dealing with this problem.

Method 1: (zero-probabilities are omitted)

With this method, the datapoints which had a frequency of suppression $P(S/r,\theta)=s_i/n_i=0$ are deleted. Consequently the following data points of the original sample of appendix A as shown in figure Fig 12 were not considered.

Caliber	No.	P(S/θ,r)	θ	r
81 mm	29	0	41	117
	30	0	36	127
	31	0	60.5	95
	32	0	46	108
	34	0	60.5	95
	35	0	46	108
	36	0	36	127
105 mm	15	0	17	104
	19	0	36	158
	23	0	46	135
	24	0	46	135
	28	0	56	108
	29	0	56	108
	31	0	71	97
155 mm	3	0	0	200
	15	0	14	209
	17	0	23	125
	18	0	14	209

Figure 12 - DELETED DATA POINTS

Method 2 and 3 (Clustering of data points)

One or more zero data points i, i.e. $\hat{P}(S/\theta,r) = 0$ $i=1,2,...m$ will be clustered with one other data point j which has a nonzero conditional frequency of suppression, i.e. $\hat{P}(S/\theta,r) \neq 0$ and which possesses the closest distance r_j and angle θ_j to the zero data point(s).

By clustering (m+1) data points together and taking the mean of the frequencies, θ and r within each cluster, we are able to replace the data points of the cluster sets. This procedure is inevitably somewhat arbitrary.

$$\hat{P}(S/\theta,r) = \frac{1}{m+1}\sum_{i=1}^{m+1}\hat{P}_i(S/\theta,r)$$

$$\bar{\theta} = \frac{1}{m+1}\sum_{i=1}^{m+1}\theta_i \qquad \bar{r} = \frac{1}{m+1}\sum_{i=1}^{m+1} r_i \tag{3.9}$$

The bars above the symbols represent the cluster average. With these averages two different replacement procedures can be applied:

Each cluster will be replaced by its cluster average. This reduces the data sample.

Each cluster point will be replaced by its cluster average. Consequently the number of data points stays unchanged.

Figure 13 lists the replaced data points of the samples of appendix A.

	Old Values				New Values					
					Method 2			Method 3		
Caliber	No.	P(S/θ,r)	θ	r	P(S/θ,r)		r	P(S/θ,r)	θ	r
81 mm	33	0.13	36	127	0.043	36	127	0.043	36	127
	30	0	36	127				0.043	36	127
	36	0	36	127				0.043	36	127
	35	0	46	108	0.033	40	110.25	0.033	40	110.25
	32	0	46	108				0.033	40	110.25
	29	0	41	117				0.033	40	110.25
	24	0.13	27	108				0.033	40	110.25
	34	0	60.5	95	0.167	60.5	95	0.167	60.7	95
	31	0	60.5	95				0.167	60.7	95
	28	0.5	60.5	95				0.167	60.7	95
105 mm	14	0.13	17	104	0.065	17	104	0.065	17	104
	15	0	17	104				0.065	17	104
	19	0	36	158	0.1	36	134	0.1	36	134
	20	0.2	37	110				0.1	36	134
	23	0	46	135	0.06	47	130	0.06	47	130
	24	0	46	135				0.06	47	130
	25	0.17	49	120				0.06	47	130
	28	0	56	108	0.03	64	102	0.03	64	102
	29	9	56	108				0.03	64	102
	30	0.13	71	97				0.03	64	102
	31	0	71	97				0.03	64	102
155 mm	3	0	0	200	0.095	0	200	0.095	0	200
	6	0.19	0	200				0.095	0	200
	9	0.38	7	202	0.13	12	207	0.13	12	207
	15	0	14	209				0.13	12	207
	18	0	14	209				0.13	12	207
	14	0.14	23	125	0.07	23	125	0.07	23	125
	17	0	23	125				0.07	23	125

Figure 13 - CHANGED DATA POINTS

Method 4 (data transformation by Cox)

This method is analogous to one suggested by D. R. Cox [Cox, p.33]. The derivation for the variable y_i shown on these pages can be applied correspondingly in the following manner:

$$\underset{\sim}{Y}_i = \ln\left(\frac{S_i + a}{n_i}\right) \qquad (3.10)$$

where the random number S_i is the number of suppressions (successes, given n_i trials).

The constant a, which represents an unbiasing adjustment, is derived in appendix B. It is taken to be:

$$a = \frac{1}{2}\,(1 - \hat{P}(S/\theta,r)) \ . \qquad (3.11)$$

This is suggested by an auxiliary analysis similar to that of Cox.

Hence the transformation results in the formula:

$$\underset{\sim}{Y}_i = \ln\left(\frac{S_i + \frac{1}{2}\,(1 - \hat{P}_i(S/\theta,r))}{n_i}\right) \qquad (3.12)$$

This formula enables one to include data points that involve zero observed frequencies of suppression; the embarassment of taking the logarithm of zero is no longer present.

Method 5 (unweighted iterative regression)

As an extention of the method discussed last, this method replaces the value of $\hat{P}(S/\theta,r)$ in the formula

$$\underset{\sim}{Y}_i' = \left(\frac{S_i + \frac{1}{2}(1 - \hat{P}_i(S/\theta,r))}{n_i}\right) \tag{3.13}$$

After each iteration by an estimate of the probability of suppression $\hat{P}(S/\theta,r)$, the number of iterations was determined by the appearance of convergence of the parameters K and ε.

The two following methods are weighted regressions which are necessitated by the earlier described heteroscedasticity.

Method 6 (weighted iterative regression $Var(Err) \sim r^4$)

This method is suggested by the fact that the observed variability in residuals increases with r. [Pindyck, p. 100] Since the error variance is not known, it is reasonable to assume the existence of a simple relationship between the error variances $Var(Err_i)$ and the values of one of the explanatory variables in the regression model. In this analysis, the distance from the foxhole to the explosion was chosen as the important explanatory variable. By using

$$Var[Err_i] \sim r_i^4 \tag{3.14}$$

the resulting regression formula is:

$$\frac{Y_i}{(Var[Err_i])^{1/2}} = \beta_1 \frac{u_i}{(Var[Err_i])^{1/2}} + \beta_2 \frac{v_i}{(Var[Err_i])^{1/2}} + \frac{Err_i}{(Var[Err_i])^{1/2}} \tag{3.15}$$

This equation can be reduced to the following regression function:

$$y'_i = \beta_1 + \beta_2 v'_i \tag{3.16}$$

where

$$Y'_i = \frac{\ell n\left(\dfrac{S_i + \frac{1}{2}(1 - \hat{P}_i(S/\theta,r))}{n_i}\right)}{r_i^2}$$

$$v'_i = \frac{v_i}{r_i^2} \tag{3.17}$$

Afterwards, the iteration procedure is equivalent to method 5. This method tends to make the variance of the residuals around the fitted line of more nearly constant variance; estimates of the model parameters should be thereby improved.

Method 7 (weighted iterative regression)

Using the error variance derived in appendix C, which has the form;

$$\mathrm{Var}[\ln f_i] = \frac{1 - \hat{P}_i(S/\theta,r)}{n_i\ \hat{P}_i(S/\theta,r)} \tag{3.18}$$

where

$$f_i = \frac{S_i}{n_i} \tag{3.19}$$

the regression formula will be modified as follows:

$$\frac{y_i}{(\mathrm{Var}[\ln f_i])^{1/2}} = \beta_1 \frac{u_i}{(\mathrm{Var}[\ln f_i])^{1/2}} + \beta_2 \frac{v_i}{(\mathrm{Var}[\ln f_i))]^{1/2}} \tag{3.20}$$

where y is again:

$$y_i = \ln\left(\frac{S_i + \frac{1}{2}(1 - \hat{P}_i(S/\theta,r))}{n_i}\right) \tag{3.12}$$

as derived in method 4. The resulting equation has the form:

$$\frac{\ln\left(\frac{S_i + \frac{1}{2}(1 - \hat{P}_i(S/\theta,r))}{n_i}\right)}{\left(\frac{1 - \hat{P}_i(S/\theta,r)}{n_i \cdot \hat{P}_i(S/\theta,r)}\right)^{1/2}} = \left\{\begin{array}{l} \beta_1 \frac{u_i}{\left(\frac{1 - \hat{P}_i(S/\theta,r)}{n_i \cdot \hat{P}_i(S/\theta,r)}\right)^{1/2}} \\ + \beta_2 \frac{v_i}{\left(\frac{1 - \hat{P}_i(S/\theta,r)}{n_i \cdot \hat{P}_i(S/\theta,r)}\right)^{1/2}} + \mathrm{Err}'_i \end{array}\right. \tag{3.21}$$

The iterative procedure in this method is performed by using the estimate of the conditional probability of suppression as input for each succeeding iteration. The number of iterations was determined as in method 5 and 6 by the apparent convergence of

the estimators of the parameters K and ε.

In order to perform the multiple linear regression, we switch over to matrix notation, where we can write the normal equations in the following form:

$$Z^T \cdot Z \cdot \hat{\beta} = Z^T \cdot Y \qquad (3.22)$$

The matrix Z is the design matrix which consists of vector U, the square of the distances and V, the product of the cosθ and the square of the distances. The variables y and β represent a (sample size 1×1) and a (1×2) vector respectively. For the experiments analyzed, the entries of the design matrix were obtained by solving the normal equations for $\hat{\beta}$. We find:

$$\hat{\beta} = (Z^T \cdot Z)^{-1} \cdot Z^T \cdot Y \qquad (3.23)$$

Remark: Capital letters used for matrix notation.

The parameters $\hat{K}$ and $\hat{\varepsilon}$ can be evaluated by using the equations:

$$\hat{K} = -\frac{1}{\hat{\beta}_1}$$

$$\hat{\varepsilon} = \hat{\beta}_2 \cdot K \qquad (3.24)$$

As a modification of the suppression function (2.1) in chapter II. B. 1 the function (2.4) in the same chapter

was considered.

$$P(S/\theta,r) = L\cdot\exp\left[-\frac{1}{K}\, r^2(1 - \varepsilon\cos\theta)\right] \qquad (2.4)$$

The inclusion of the parameter L allows for the possibility that suppression may not occur for shots that fall in very close proximity to the foxhole.

Method 8 (similar to method 4 but using model above)

In this method the regression procedure and the method 4 were applied to:

$$Y_i = \beta_1 u_i + \beta_2 v_i + \beta_3 + Err_i \qquad (3.25)$$

where

$$y_i = \ell n\left(\frac{S_i + \frac{1}{2}(1 - \hat{P}_i(S/\theta,r))}{n_i}\right)$$

$$u_i = r_i^2$$

$$v_i = r_i^2 \cos\theta \qquad (3.26)$$

$$\beta_1 = -\frac{1}{K}$$

$$\beta_2 = \frac{\varepsilon}{K}$$

$$\beta_3 = e^L$$

The normal equations have the same form as before:

$$\hat{\beta} = (Z^T\cdot Z)^{-1}\cdot Z^T\cdot Y \qquad (3.23)$$

but Z, the design matrix consists this time of the vectors U,V, and a column vector consisting only of ones. The resulting $\hat{\beta}$ vector consists now of 3 elements, from which it is possible to evaluate the different parameters K, ε, and L.

The calculation of the parameters for all methods was performed for each type of ammunition separately by applying the computer language APL. The programs which where used for this regression are listed in appendix I. The appendixes D through G show the analysis of the residuals after the regression of the original data of appendix A.

The analysis for each ammunition type was performed according to the same pattern, and includes the following steps.

For each earlier developed method (1 through 7):

Determination of BETA(1) and BETA(2)

Determination of K

Determination of ε

Plot of the residuals as a function of y as defined in each method earlier. The APL function which was used has the name SCAT and belongs to the library package OA 2 3660 (available at W. R. Church Computer Center).

Plot of the residuals of the regression as a single array with the function BOXPLOT of the same library package. The plot characterizes the quartiles, the interquartile distance, the median, data points inside and outside the 1 and 1.5 interquartile distance and outliers [McNeil, p.13 and 71,72].

Numerical values of the residuals.

Histograms for the residuals of the regression which show the relative frequency and statistical features.

For method 8, which is an analysis based on different assumptions for the probability of suppression $P(S/\theta,r)$, the same analysis was performed, with the exception of the histograms for residuals.

The methods 1-7 describe the attempt, to master the problem of zero-probabilities in the original data set as well as the problem of a possible heteroscedastisity.

For the later methods, in which more appropriate statistical tools were used, the systematic structure of the residuals seems to disappear up to a certain point. The residuals concentrate themselves more and more symmetrically around their mean and median (compare particular the boxplots and the histograms in the appendixes D, E, F, and G).

The relative large remaining range of the residuals is determined by single outliers. The analysis showed differences for different kinds of ammunition. Among the three ammunitions considered, the analysis of 81 mm showed the smallest spread for the residuals, for almost all regression methods.

The appendix H shows for the iterative regression methods 5,6, and 7 the develcpment of K and ε for the different kinds of ammunition. In each method , a convergence of the values K and ε with increasing iteration can be observed. The starting value for K is in method 5 smaller and in method 6 and 7 larger than its corresponding value after convergence is obtained. This is true for all types of ammunition. An equivalent observation can be made for the ε-values.

A graph for the iterating ε for method 5, 155 mm could not be plotted because of the very small change of the ε-values along each iteration.

The third page of appendix H displays the numerical values for K and ε produced in each method for the three different types of ammunition.

The analysis of the parameters K and ε showed the following result;

For the iterative methods (5,6,7) the K and ε-values approached with increasing iteration a limit value (see page one and two of appendix H); this occured after about the sixth iteration. The final values for these methods are also presented in the tableau of appendix H (third page). The plots for K and ε show different shapes for different methods and partly also for different types of ammunitions . The approach to the final value may occur from a relative small or a relative high value. Among the three ammunition types considered, the analysis of 81 mm showed the smallest spread for the residuals, for almost all regression methods. The scale parameter K influences the probability of suppression $P(S/\theta,r)$ as stated under formula (2.1) in chapter II.B.1. in the following manner:

An increasing value of K leads to an increase of the probablility of suppression $P(S/\theta,r)$ .It is reasonable to assume that the suppression effect increases with the increase of the caliber. This behavior was confirmed by the data analysis; except for method one and three where an inversion could be observed between the K-values for 81 mm, 105 mm, and 155 mm and for 81 mm and 105 mm respectively.

This distortion results from the fact that in method one all data points with probabilities of suppression equal to zero are disregarded and in method three the cluster procedure emphasizes the average values produced by the clustering. We prefer the latter methods.

Contrary to these observations on the K- values, a general trend for the ε-values can not be related to different methods or different kinds of ammunition. The most reliable value of K and ε seems to be found by method

7. This method considers the different variabilities for different sets of distance and angle.

In the course of this paper, the value of K and ε derived in method 7 are taken as input for developing the rates for the model of section II.B.

The third parameter L developed by method 8 is valid only up to a probability of suppression evaluation equal to 1.0. Since for 81 mm and 155 mm L is larger than one, the function for the model should be decomposed. This alternative approach to model suppression is beyond the scope of this paper and will not be considered further at this point, although it is certainly a topic worthy of further research.

As a possible further step, confidence intervals for K and ε could be developed.

2. Suppression Time

The suppression time data, which were collected in seconds, represent the time for which an individual remains suppressed as a reaction of a single round. During this time, the individual was unable to carry out his mission; he is in hiding in an effort to survive. According to the setup of the experiment, it was possible for the suppressee to react to detonations, which he was able to observe and to hear or to hear only (visual/auditory and auditory perceptors). This is consistent with the model design in chapter II.

This analysis is an attempt to compare the suppression interval gained from the experiment with a certain distribution whose parameters are estimates of the data. The GAMMA distribution was selected, because the range of the random variable X, representing the time, is limited below by zero and by $+\infty$ above.

The data of suppression interval for the calibers 81 mm, 105 mm, and 155 mm are plotted in histograms in appendix J.

The estimates $\hat{\lambda}$ and $\hat{r}$ for the Gamma distribution are derived from the expectation and the variance of the data.

$$\hat{\lambda} = \frac{E[x]}{Var[x]} = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i}{\frac{1}{n}\sum_{i=1}^{n} (x_i-\bar{x})^2} \qquad \hat{r} = \hat{\lambda} \cdot E[x] = \frac{(\sum_{i=1}^{n} x_i)^2}{n \sum_{i=1}^{n} (x_i-\bar{x})^2} \tag{3.27}$$

With these parameters, the Gamma density can be computed in the considered range, by the formula:

$$f(x/\hat{\lambda},r) = \begin{cases} \frac{\hat{\lambda}^{\hat{r}}}{\Gamma(\hat{r})} x^{\hat{r}-1} e^{-\hat{\lambda}x} & 0 \leq x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{3.28}$$

For comparison, the related density was computed by

$$f(x) = \frac{\text{frequency in interval}}{N \cdot \text{interval}} \tag{3.29}$$

within the same range, where the frequency in a particular interval and the interval itself is taken from the histograms of appendix J, and the constant N is the sample size of the measured suppression intervals.

The numerical values for the estimates $\hat{\lambda}$ and $\hat{r}$ and the values for the sample size N, the interval Δx, and the range are displayed in the following table.

Caliber	$\hat{\lambda}$	$\hat{r}$	N	INTERVAL Δx	RANGE
81 mm	0.686	2.174	445	0.5	0 to 16.15
105 mm	0.468	1.974	348	0.4	0 to 13.7
155 mm	0.587	2.591	101	0.5	0 to 16.75

The following Fig 14 compares the Gamma density (3.28) with the height of the related estimated probability of the data (3.29). The sketches show that the Gamma density for each ammunition type respectively underestimates the related frequency of the data because of its long tails. In fact, any distribution which meets the requirements above could be taken for a comparison, although the Gamma distribution seems to be a particularly good choice for the 81 mm caliber ammunition.

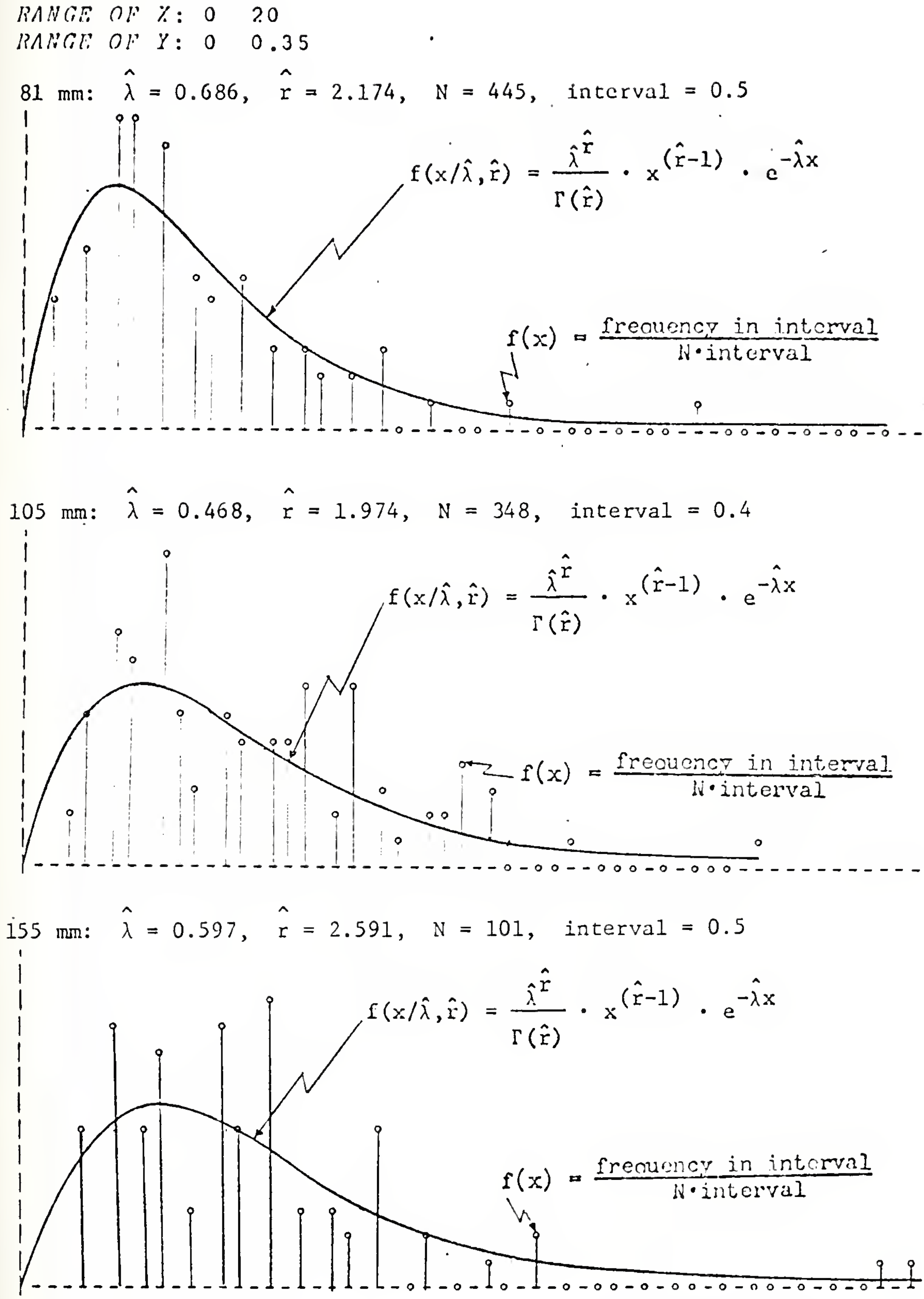


Figure 14 - COMPARISON GAMMA DISTRIBUTION AND DATA

In order to intensify the comparison, as a next step the skewness and the kurtosis of the data and of the Gamma distribution with the estimates above will be contrasted.

These characteristics express the symmetry of the distribution about its point of central tendency and the relative concentration of cases at the center and along the tails of the distribution.

For the derivation of the skewness and the kurtosis for a Gamma distribution, we need the first four moments about the origin:

$$\mu_1 = E[x] = \frac{r}{\lambda} \qquad \mu_2 = E[x^2] = \int_0^\infty x^2 e^{-\lambda x} \frac{(\lambda x)^{r-1}}{\Gamma(r)} \lambda \, dx \tag{3.30}$$

By changing the variable of integration

$$z = \lambda x \qquad dz = \lambda \, dx \tag{3.31}$$

we find

$$\mu_2 = E[x^2] = \int_0^\infty \frac{z^2}{\lambda^2} e^{-z} z^{r-1} \frac{1}{\Gamma(r)} \lambda \cdot \frac{1}{\lambda} dz \tag{3.32}$$

$$= \frac{\Gamma(r+2)}{\lambda^2 \Gamma(r)} \int_0^\infty z^{T+2-1} e^{-z} \frac{1}{\Gamma(r+2)} dz$$

but the above integral is equal to one, hence

$$\mu_2 = \frac{1}{\lambda^2} (r + 1)r \tag{3.33}$$

In a similar fashion, the third and the fourth moment can be derived.

$$\mu_3 = \frac{1}{\lambda^3} (r + 2)(r + 1)r \qquad \mu_4 = \frac{1}{\lambda^4} (r + 3)(r + 2)(r + 1)r \tag{3.34}$$

Converting these moments to moments about the mean by using binomial expansion:

$$\tilde{\mu}_3 = \mu_3 - 3\mu_2 E[x] + 2(E[x])^3 \tag{3.35}$$

$$\tilde{\mu}_4 = \mu_4 - 4\mu_3 E[x] + 6\mu_2 (E[x])^2 - 3(E[x])^4$$

we can compute the skewness and the kurtosis

$$\alpha_3 = \frac{\tilde{\mu}_3}{\sigma^3} \qquad \alpha_4 = \frac{\tilde{\mu}_4}{\sigma^4} \tag{3.36}$$

For a Gamma distribution we get the following formulas.

$$\alpha_3 = \left[\frac{(r+2)(r+1)r}{\lambda^3} - 3\,\frac{(r+1)r}{\lambda^2} \cdot \frac{r}{\lambda} + 2\,\frac{3}{\lambda^3} \right] \cdot \frac{1}{(r/\lambda^2)^{3/2}}$$

$$\alpha_3 = \frac{2}{\sqrt{r}}$$

and by similar operation:

$$\alpha_4 = \frac{3r + 6}{4} \tag{3.37}$$

Using the above estimates for λ and r we get:

$$\alpha_3 = \frac{2}{\sqrt{\hat{r}}} \qquad \alpha_4 = \frac{3\hat{r} + 6}{\hat{r}} \tag{3.38}$$

The formulas for the skewness and kurtosis derived from the data itself are:

$$\hat{\alpha}_3 = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^3}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^{3/2}}$$

$$\hat{\alpha}_4 = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^4}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^{2}} \tag{3.39}$$

The resulting numerical values for the skewness and kurtosis derived for the Gamma distribution and from the data are displayed in the following table:

	81 mm	105 mm	155 mm
Skewness α_3 GAMMA	1.356	1.423	1.242
Skewness $\hat{\alpha}_3$ DATA	2.156	1.032	2.103
Kurtosis α_4 GAMMA	5.759	6.039	5.315
Kurtosis $\hat{\alpha}_4$ DATA	8.004	1.309	6.750

In discussing the numerical features we can see that the skewness of the Gamma distribution and of the data itself shows an asymmetric right-skewness (positive values). Although the values differ considerably . So is e. g. the skewness of the data for 155 mm almost twice as big as the skewness based on the Gamma distribution.

The same can be said for the kurtosis. There is significant difference between their numerical values.

This analysis supports the fact that the Gamma distribution can only be a rough fit to the data given, and as already earlier expressed, a fit of another distribution probably would have been as successful as this one.

C. VERIFICATION OF THE MODEL

In order to supply general features as input factors for the decision process for military leadership, the purpose of this paragraph is to compute numerical values for the probabilities and rates developed in chapter II.B.1. and 2.

For the computation of numerical values, it is necessary to make the following reasonable assumptions:

It is assumed that an artillery unit consist of six weapons all either 105 mm or 155 mm, or that three 81 mm launchers are combined to a mission unit. Hence the fire rate λ_f for such units will be concluded to be [Wiener, p.189, 211, 213]:

81 mm unit with 50, 55, and 60 rounds/min

105 mm unit with 28, 32, and 36 rounds/min

155 mm unit with 18,21, and 24 rounds/min.

The standard deviation σ for the density for artillery hits will be considered to be [FM 6-161-1, p.53]:

25,30,and 35 m for 81 mm unit

20,25,and 30 m for 105 mm unit

30,35,and 40 m for 155 mm unit.

The numbers are taken from the field manual 6-161-1 and out of working papers of CDEC. They are rounded for convenience.

The parameter H of the probability of kill is chosen completely arbitrarily with 100 for 81 mm, 150 for 105 mm, and 200 for 155 mm and has nothing to do with experimentally observed values for the weapons here in question.

The parameter estimates K and ε are taken from appendix H (tableau) with the values:

	81 mm	105 mm	155 mm
$\hat{k}$ =	1360.498	1991.151	2349.845
$\hat{\varepsilon}$ =	0.871	0.769	0.359

The following figure displays the conditional probability of suppression with the above estimators for the parameters.

$$P(S/\theta,r) = \exp\left[-\frac{1}{\hat{K}} r^2(1 - \hat{\varepsilon}\cos\theta)\right] \tag{3.40}$$

for θ=0 i.e. along the main direction of sight.

$$P(S/\theta = 0°,r) = \exp\left[-\frac{1}{\hat{K}} r^2(1 - \hat{\varepsilon})\right] \tag{3.41}$$

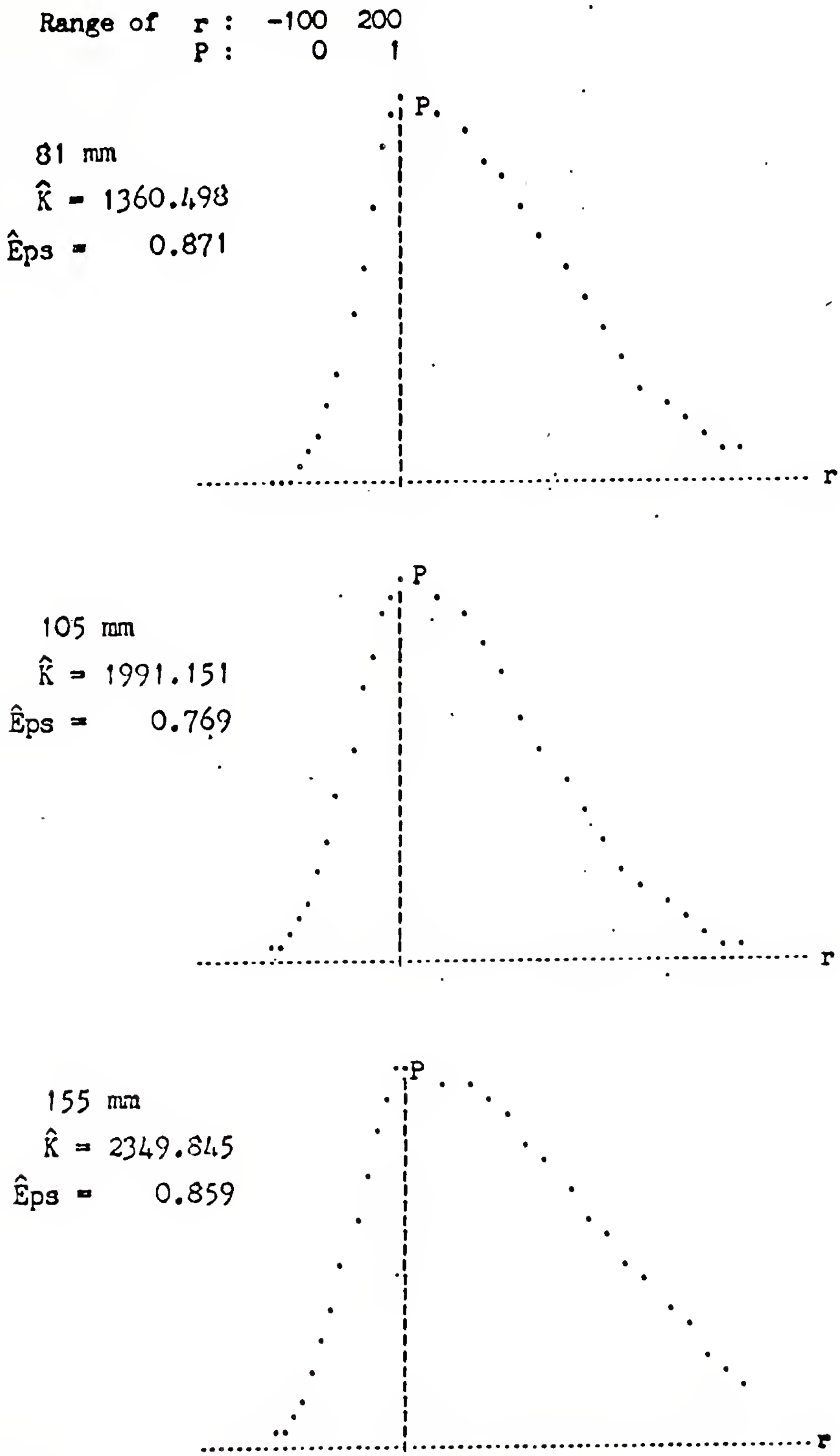


Figure 15 - FUNCTION $P(S/\theta = 0°, r) = \exp[-\frac{1}{K} r^2 (1 - \varepsilon)]$

For the above selected values of the standard deviation σ, the fire rate $\hat{\lambda}_f$, and the parameters $\hat{K}$, $\hat{\varepsilon}$, and H the values of the probabilities

$$\hat{P}(S) = \frac{\hat{K}}{2\sigma^2 \sqrt{\left(1 + \frac{\hat{K}}{2\sigma^2}\right)^2 - \hat{\varepsilon}^2}} \tag{3.41}$$

$$\hat{P}_K(S) = P(\hat{S}) - \frac{\hat{K}}{2\sigma^2 \sqrt{\left(1 + \frac{\hat{K}}{2\sigma^2} + \frac{\hat{K}}{H}\right)^2 - \hat{\varepsilon}^2}} \tag{3.42}$$

$$\hat{P}(K) = \frac{1}{\sigma^2/H + 1} \tag{3.43}$$

and rates

$$\begin{aligned} \hat{\lambda}_s(\text{with no killing}) &= \lambda_f \hat{P}(S) = \\ \hat{\lambda}_s(\text{with killing}) &= \lambda_f \hat{P}_K(S) \\ \hat{\lambda}_K &= \lambda_f \hat{P}(K) \end{aligned} \tag{3.44}$$

are displayed on the succeeding two figures number 16 and 17. Numerical values for the rate of rise λ_u are also computed and displayed in figure 17. The collected data (suppression intervals) were not distinguished among individuals as in the course of the foregoing analysis. Because of this fact the numerical values for λ_u are based on the expectation of duration time with regard to population.

$$E[\tau] = \frac{1}{\lambda_f P(S)} \left[\int_0^\infty e^{\lambda_f P(S) t} f_p(t)dt - 1 \right] \tag{3.45}$$

It might be worthwhile to evaluate the rate of rise by applying the Gamma distributions for $f_p(t)$ which were discussed in chapter III.B.2, and contrast them to the values of λ_u gained by the data itself.

Hence solving equation (3.45) by having set

$$f_p(t) = \frac{e^{-\lambda t}}{\Gamma(r)} (\lambda t)^{r-1} \lambda \tag{3.46}$$

we find:

$$E[\tau] = \frac{1}{\lambda_f P(S)} \left[\int_0^{\infty} e^{\lambda_f P(S) t} \frac{e^{-\lambda t}}{\Gamma(r)} (\lambda t)^{r-1} \lambda \, dt - 1 \right] \tag{3.47}$$

if $\lambda > \lambda_f P(S)$ and by changing the variable of integration:

$$(\lambda - \lambda_f P(S))t = z \qquad (\lambda - \lambda_f P(S))dt = dz \tag{3.48}$$

we get:

$$E[\tau] = \frac{\lambda^r}{\lambda_f P(S)[\lambda - \lambda_f P(S)]^r} \underbrace{\int_0^{\infty} \frac{e^{-z} z^{r-1}}{\Gamma(r)} dz}_{=1} - \frac{1}{\lambda_f P(S)}$$

$$E[\tau] = \frac{\lambda^r}{\lambda_f P(S)[\lambda - \lambda_f P(S)]^r} - \frac{1}{\lambda_f P(S)} \tag{3.49}$$

By using the earlier developed estimators, we receive the mathematical expression for the rate of rise based on a Gamma distribution for the suppression interval data.

$$\hat{\lambda}_u = \left[\frac{\hat{\lambda}^r}{\lambda_f \hat{P}(S)[\hat{\lambda} - \lambda_f \hat{P}(S)]^{\hat{r}}} - \frac{1}{\lambda_f \hat{P}(S)} \right]^{-1} \tag{3.50}$$

Remark: This derivation is only true for $\lambda > \lambda_f P(S)$, which means that for applying this formula, the fire rate λ_f may not exceed the value:

$$\lambda_f \leq \frac{\lambda}{P(S)} \tag{3.51}$$

Otherwise the integral (3.47) explodes towards infinite and the resulting rate of rise would be

$$\lambda_u = 0 \tag{3.52}$$

which is consistent within the set of assumptions.

Instead of using the Gamma distribution the data themselves were applied in a second step to calculate an estimate for $E[\tau]$ and λ_u based on the number of observations n.

$$\hat{E}[\tau] = \frac{1}{\lambda_f P(S)} \left[\left(\frac{1}{n} \sum_{i=1}^{n} e^{\lambda_f P(S) t_i} \right) - 1 \right] \tag{3.53}$$

The approximate value for the rate of rise is then

$$\hat{\lambda}_u = \frac{1}{\hat{E}[\tau]} \tag{3.54}$$

If killing as an additional event is considered, the rate of rise is computed with the same formulas (3.49) and (3.53) except that this time $P_K(S)$ is used instead of $P(S)$. The computation of the values for the formulas (3.50) and (3.54) was performed by APL and FORTRAN respectively. The programs are displayed on appendix K.

PROBABILITIES P(S), $P_k(S)$, AND P(K)

81 mm — σ / P	25 m	30 m	35 m
P(S)	0.573	0.495	0.430
$P_k(S)$	0.503	0.446	0.394
P(K)	0.137	0.100	0.075

105 mm — σ / P	20 m	25 m	30 m
P(S)	0.731	0.643	0.564
$P_k(S)$	0.582	0.542	0.492
P(K)	0.272	0.193	0.142

155 m — σ / P	30 m	35 m	40 m
P(S)	0.573	0.497	0.432
$P_k(S)$	0.480	0.428	0.378
P(K)	0.181	0.140	0.111

Figure 16 - NUMERICAL VALUES FOR P(S), $P_k(S)$, AND P(K)

Rates	81 mm				105 mm				155 mm				row
	λf / sec	σ 25m	σ 30m	σ 35m	λf / sec	σ 20m	σ 25m	σ 30m	λf / sec	σ 30m	σ 35m	σ 40m	
Rate of Suppression $\hat{\lambda} S$ (without killing)	0.83 0.92 1	0.475 0.527 0.573	0.411 0.455 0.495	0.357 0.395 0.430	0.46 0.53 0.60	0.336 0.387 0.438	0.295 0.341 0.385	0.259 0.298 0.338	0.3 0.35 0.4	0.172 0.200 0.229	0.149 0.174 0.193	0.129 0.151 0.172	(1)
Rate of Suppresion $\hat{\lambda} S$ (with killing)	0.83 0.92 1	0.417 0.462 0.503	0.370 0.410 0.446	0.327 0.362 0.394	0.46 0.53 0.60	0.267 0.308 0.349	0.249 0.287 0.325	0.226 0.260 0.295	0.3 0.35 0.4	0.144 0.168 0.192	0.123 0.149 0.171	0.113 0.132 0.151	(2)
Rate of killing $\hat{\lambda} K$	0.83 0.92 1	0.113 0.126 0.137	0.083 0.092 0.100	0.062 0.069 0.075	0.46 0.53 0.60	0.125 0.144 0.163	0.088 0.102 0.115	0.065 0.075 0.085	0.3 0.35 0.04	0.054 0.063 0.072	0.042 0.049 0.056	0.033 0.038 0.044	(3)
Rate of Rise $\hat{\lambda} u$ based on GAMMA (without killing)	0.83 0.92 1	0.039 0.023 0.012	0.065 0.047 0.033	0.091 0.072 0.057	0.46 0.53 0.60	0.030 0.012 0.002	0.048 0.028 0.013	0.066 0.046 0.029	0.3 0.35 0.4	0.122 0.106 0.091	0.135 0.121 0.107	0.146 0.134 0.121	(4)
Rate of Rise $\hat{\lambda} u$ based on GAMMA (with killing)	0.83 0.92 1	0.062 0.044 0.030	0.084 0.066 0.050	0.106 0.088 0.073	0.46 0.53 0.60	0.062 0.043 0.025	0.071 0.052 0.035	0.084 0.065 0.048	0.3 0.35 0.4	0.138 0.124 0.111	0.147 0.134 0.122	0.156 0.145 0.134	(5)
Rate of Rise $\hat{\lambda} u$ based on data (without killing)	0.83 0.92 1	0.024 0.013 0.007	0.045 0.029 0.019	0.071 0.052 0.038	0.46 0.53 0.60	0.059 0.042 0.030	0.074 0.057 0.043	0.089 0.073 0.058	0.3 0.35 0.4	0.117 0.100 0.084	0.131 0.116 0.101	0.144 0.130 0.117	(6)
Rate of Rise $\hat{\lambda} u$ based on data (with killing)	0.83 0.92 1	0.043 0.027 0.017	0.064 0.045 0.032	0.088 0.068 0.052	0.46 0.53 0.60	0.086 0.069 0.054	0.094 0.077 0.063	0.104 0.089 0.074	0.3 0.35 0.4	0.134 0.119 0.105	0.144 0.131 0.117	0.154 0.142 0.130	(7)

Figure 17 - RATES ?OR THE MOCEL

Row (4) and (6) and row (5) and (7) of figure 17 display comparable values for the rate of rise. As it was stated earlier, the Gamma distribution underestimates the related frequency of the data and hence the comparable values for the rate of rise λ_u in row (4) are larger than the values in row (6). The same is true for row (5) and (7). They differ in general by 10 - 12%. If we are willing to live with this fact, the rates of rise evaluated by the fitted Gamma distribution are a good approximation for the values computed by the data.

In order to verify the four Lanchester equations (2.76), (2.77), (2.98), and (2.99), presented in chapter II.C., it is necessary to specify particular rates given in the figure before. In case of a certain known composition of fire units, specific rates could be developed as inputs for the model equations.

By this, the model equations receive their specific shape and scale, their general behavior remains the same, as can be seen in figure 9 and 10.

IV. DISCUSSION

Based on the research effort of this paper it was found that the phenomenon of suppression as defined earlier, is a multidimensional problem. It is influenced by psychological, physiological, and environmental variables. There exist many possibilities to model dependencies in general form, however to make quantitative statements about suppression the modellers have to restrict their efforts to those variables which are observable and measurable. Since the main objective of this paper has been to establish models which are able to express relationships quantitatively, the main thrust has been to formulate suppression as functions of weapon systems and their dispersions.

For the evaluations of the models, a set of simplifying assumptions was necessary in order to guarantee a mathematical transparency. The dependencies developed in this thesis postulate some satisfying results in modelling suppression. The models reflect sufficient accuracy of:

> The physionomy of the human being and its resulting behavior with regard to suppression. Suppression is mainly caused by visual and auditory perception.
>
> The influence of the weapon systems i.e. their size and their firing capability are determining factors for the amount of suppression.

Of course the detailed results, i.e. the estimation of model parameters, are based on selected weapon systems and scenarios. Many possibilities exist for further work in this area particularly under the aspect of including extended human factors components in the construction of

suppression models.

The authors feel the paper may provide a contribution to future design of wargames and simulations as well as weapon systems. It also supports a more careful analysis of the combat situation.

APPENDIX A

ORIGINAL DATA 81 mm

#	θ in degree	Range(r) in meter	P(S/θ,r)	# of Trials	# of Success
1	0	30	1	27	27
2	0	75	0.5	32	16
3	0	90	0.61	32	20
4	0	30	1	27	27
5	0	60	0.81	32	26
6	0	90	0.69	32	22
7	0	30	0.94	27	25
8	0	60	0.69	32	22
9	0	90	0.44	32	14
10	35	42	0.85	27	23
11	17	88	0.55	32	18
12	14	95	0.76	32	24
13	35	42	0.63	27	17
14	21	67	0.5	32	16
15	14	95	0.21	32	7
16	35	42	0.38	27	10
17	21	67	0.21	32	7
18	14	95	0.17	32	5
19	52	67	0.57	27	15
20	31	96	0.57	32	18
21	27	108	0.23	32	7
22	52	67	0.56	27	15
23	36	85	0.13	32	4
24	27	108	0.13	32	4
25	52	67	0.25	27	7
26	36	85	0.06	32	2
27	27	108	0.06	32	2
28	60.5	95	0.5	27	14
29	41	117	0	32	0
30	36	127	0	32	0
31	60.5	95	0	27	0
32	46	108	0	32	0
33	36	127	0.13	32	4
34	60.5	95	0	27	0
35	46	108	0	32	0
36	36	127	0	32	0

APPENDIX A

ORIGINAL DATA 105 mm

#	θ in degree	Range(r) in meter	P(S/θ,r)	# of Success	# of Trials
1	13	133	0.3	5	16
2	0	130	0.36	6	16
3	0	100	0.43	14	32
4	0	100	0.44	14	32
5	0	80	0.4	6	14
6	0	80	0.36	5	14
7	0	60	0.88	28	32
8	0	60	0.75	24	32
9	0	35	1	45	45
10	0	35	1	45	45
11	25	143	0.25	4	16
12	31	117	0.13	4	32
13	31	117	0.13	4	32
14	17	104	0.13	4	32
15	17	104	0	0	32
16	22	85	0.27	4	14
17	28	67	0.58	19	32
18	28	67	0.29	9	32
19	36	158	0	0	16
20	37	110	0.2	3	14
21	42	46	0.71	32	45
22	42	46	0.37	17	45
23	46	135	0	0	32
24	46	135	0	0	32
25	49	120	0.17	2	14
26	47	85	0.25	8	32
27	47	85	0.13	4	32
28	56	108	0	0	32
29	56	108	0	0	32
30	71	97	0.13	6	45
31	71	97	0	0	45
32	63	69	0.38	17	45
33	63	69	0.25	11	45

APPENDIX A

ORIGINAL DATA 155 mm

#	θ in degree	Range(r) in meter	P(S/ə,r)	# of Trials	# of Success
1	0	70	0.93	32	30
2	0	110	0.56	32	18
3	0	200	0	32	0
4	0	70	1	32	32
5	0	110	0.44	32	14
6	0	200	0.19	32	6
7	18	76	0.67	32	21
8	12	114	0.38	32	12
9	7	202	0.38	32	12
10	18	76	0.71	32	23
11	12	114	0.5	32	16
12	7	202	0.25	32	8
13	33	92	0.69	32	22
14	23	125	0.14	32	4
15	14	209	0	32	0
16	33	92	0.44	32	14
17	23	125	0	32	0
18	14	209	0	32	0
19	43	114	0.57	32	18
20	43	114	0.125	32	4

APPENDIX A

SUPPRESSION INTERVALS IN SECONDS

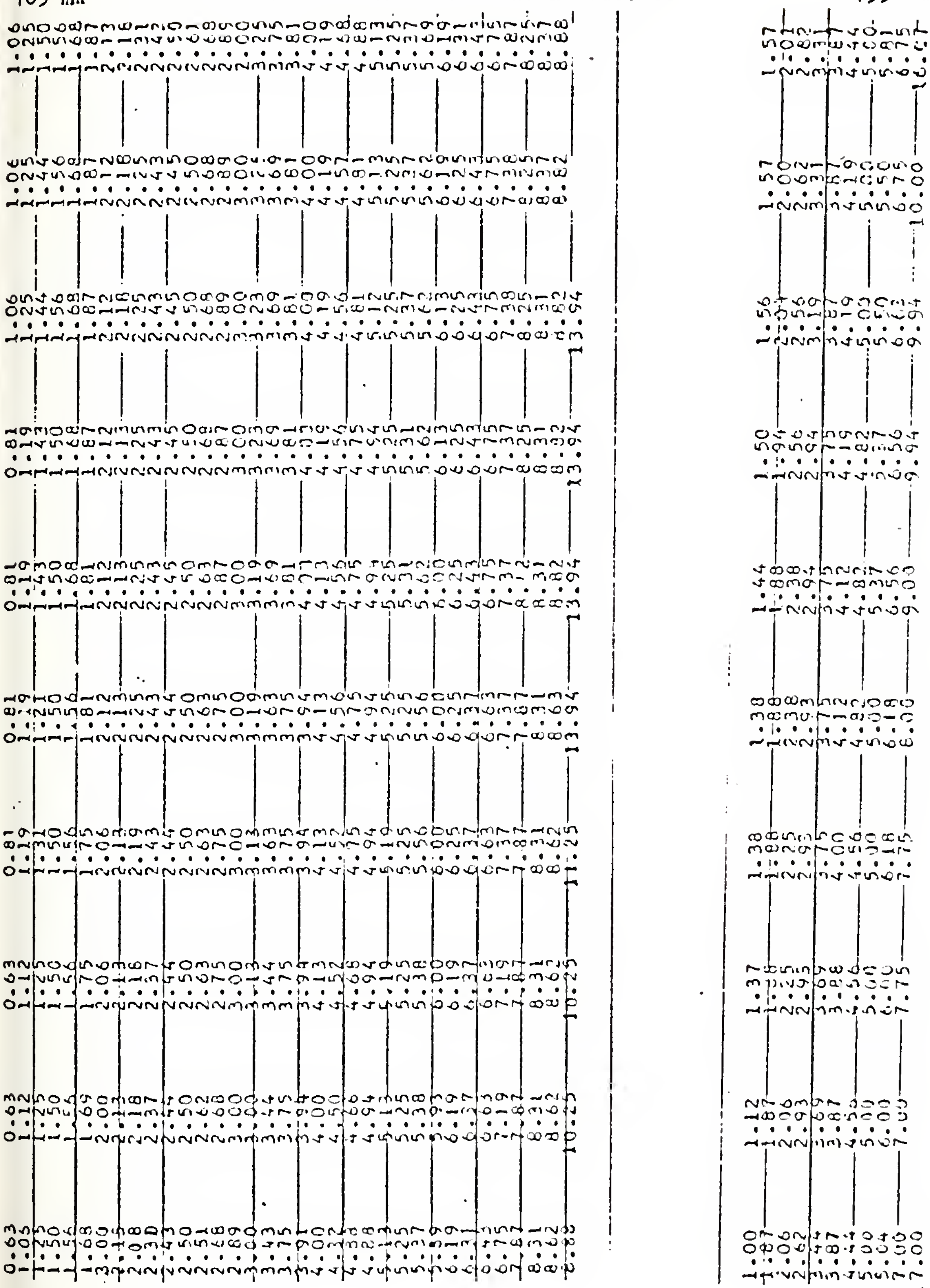
81 mm

0.37	0.37	0.43	0.43	0.50	0.50	0.50	0.56	0.56	0.56
0.62	0.68	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.81
0.81	0.80	0.81	0.81	0.82	0.82	0.82	0.94	0.94	0.94
0.94	0.94	0.94	1.00	1.00	1.00	1.06	1.06	1.06	1.12
1.12	1.06	1.06	1.18	1.19	1.19	1.00	1.19	1.19	1.19
1.19	1.19	1.25	1.25	1.25	1.31	1.31	1.31	1.31	1.31
1.31	1.31	1.31	1.31	1.31	1.31	1.38	1.38	1.38	1.44
1.44	1.50	1.50	1.50	1.50	1.50	1.56	1.56	1.56	1.56
1.56	1.56	1.56	1.56	1.56	1.56	1.57	1.57	1.63	1.63
1.63	1.69	1.63	1.65	1.65	1.68	1.68	1.69	1.69	1.65
1.69	1.69	1.65	1.75	1.75	1.75	1.75	1.75	1.75	1.75
1.75	1.75	1.80	1.81	1.81	1.81	1.81	1.81	1.81	1.81
1.81	1.81	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
1.87	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88
1.88	1.88	1.88	1.93	1.93	1.94	1.94	1.94	1.94	1.94
1.94	2.00	2.00	2.00	2.06	2.06	2.06	2.06	2.06	2.06
2.06	2.06	2.06	2.06	2.13	2.13	2.13	2.13	2.13	2.13
2.18	2.18	2.18	2.19	2.19	2.19	2.25	2.25	2.25	2.25
2.26	2.26	2.31	2.31	2.31	2.31	2.31	2.31	2.37	2.37
2.37	2.37	2.37	2.37	2.37	2.37	2.38	2.38	2.44	2.44
2.44	2.44	2.44	2.44	2.44	2.44	2.44	2.44	2.44	2.44
2.44	2.50	2.50	2.50	2.50	2.50	2.50	2.56	2.56	2.62
2.62	2.62	2.68	2.63	2.69	2.69	2.69	2.69	2.69	2.69
2.75	2.75	2.75	2.75	2.75	2.75	2.75	2.75	2.81	2.81
2.81	2.81	2.81	2.81	2.81	2.81	2.82	2.82	2.82	2.82
2.87	2.87	2.87	2.87	2.87	2.87	2.88	2.83	2.88	3.00
3.00	3.00	3.06	3.06	3.02	3.12	3.12	3.13	3.13	3.13
3.19	3.09	3.19	3.19	3.19	3.25	3.25	3.25	3.29	3.25
3.25	3.25	3.25	3.25	3.31	3.31	3.37	3.37	3.37	3.37
3.37	3.37	3.38	3.38	3.50	3.50	3.50	3.56	3.56	3.56
3.56	3.56	3.63	3.63	3.63	3.69	3.68	3.75	3.75	3.75
3.67	3.57	3.87	3.87	3.87	3.87	3.87	3.88	3.88	3.88
3.93	3.93	3.93	3.94	3.94	3.94	3.94	3.94	3.94	3.94
3.94	3.94	4.00	4.06	4.06	4.06	4.12	4.13	4.13	4.19
4.19	4.19	4.19	4.19	4.25	4.27	4.27	4.31	4.31	4.32
4.32	4.32	4.42	4.48	4.44	4.44	4.44	4.63	4.63	4.75
4.75	4.75	4.79	4.81	4.81	4.81	4.87	4.87	4.87	4.93
4.93	4.93	4.93	4.93	4.93	4.93	4.93	4.94	4.94	5.00
5.00	5.00	5.06	5.12	5.19	5.19	5.31	5.31	5.44	5.44
5.44	5.50	5.50	5.50	5.56	5.56	5.56	5.62	5.62	5.62
5.87	5.94	5.94	6.19	6.19	6.19	6.25	6.25	6.25	6.31
6.21	6.31	6.43	6.43	6.56	6.56	6.56	6.62	6.62	6.51
6.81	6.81	6.81	6.81	6.87	6.87	7.19	7.19	7.46	7.49
7.56	7.82	7.62	7.82	9.00	9.00	9.00	9.19	9.19	9.19
12.75	12.75	12.75	16.44	16.44					

APPENDIX A

SUPPRESSION INTERVALS IN SECONDS

105 mm | 155 mm



APPENDIX B

DERIVATION OF CONSTANT A

The constant a can be determined correspondingly to the following derivation:

Modifying equation (3.10) in Cox, The Analysis Of Binary Data, p. 33, we can define a transform as:

$$\hat{y}_i = \ell n \left(\frac{S_i + a}{n_i} \right) \qquad (1)$$

Starting with the original model

$$p_i = P_i(S/\theta, r) = e^{f(\theta_i, r_i)}$$

and using the log-transformation

$$y_i = \ell n \; p_i \qquad (2)$$

we subtract

$$\hat{y}_i - y_i = \ell n \left(\frac{S_i + a}{n_i} \right) - \ell n \; p_i$$

and choose the constant a such that

$$E[\hat{y}_i - y_i] = 0 \qquad (3)$$

Since $S \sim$ Binomial $(n_i p_i, \; n_i p_i (1 - p_i))$

which can be approximated by

$$S_i = n_i P(S/\theta, r) + \sqrt{n_i} \; U$$

where U is a random variable with

$$E[U] = 0$$

$$E[U^2] = p_i (1-p_i)$$

Inserting S in equation (3)

$$E[\hat{y}_i - y_i] = E\left[\ln\left(\frac{S_i + a}{n_i}\right) - \ln p_i\right]$$

$$= E\left[\ln \frac{n_i p_i + \sqrt{n_i}\, U + a}{n_i p_i}\right]$$

$$= E\left[\ln\left(1 + \left(\frac{U}{\sqrt{n_i}\, p_i} + \frac{a}{n_i p_i}\right)\right)\right]$$

Approximating this by Taylor series

$$= E\left[\left(\frac{U}{\sqrt{n_i}\, p_i} + \frac{a}{n_i p_i}\right) - \frac{1}{2}\left(\frac{U}{\sqrt{n_i}\, p_i} + \frac{a}{n_i p_i}\right)^2\right.$$

$$\left. + \frac{1}{3}\left(\frac{U}{\sqrt{n_i}\, p_i} + \frac{a}{n_i p_i}\right)^3 - \cdots + \right]$$

It is sufficient to consider the first two terms of the Taylor expansion since U and a are small in comparison to n.

$$= E\left[\frac{U}{\sqrt{n_i}\, p_i} + \frac{a}{n_i p_i} - \frac{U^2}{2 n_i p_i} - \frac{Ua}{n_i^{3/2} p_i^2} - \frac{a^2}{2 n_i^2 p_i^2}\right]$$

This equation has to be zero according to (3)

Hence

terms of order $\frac{1}{\sqrt{n_i}}$: $E[U] = 0$

terms of order $\frac{1}{n_i}$: $E\left[\frac{a}{p_i} - \frac{U^2}{2p_i^2}\right] = 0$

$$a - \frac{1}{2}(1 - p_i) = 0$$

$$a = \frac{1}{2}(1 - p_i)$$

terms of higher order are neglected.

APPENDIX C

DERIVATION OF VARIANCE

The weighting factor Var(ln f) for method 7 in chapter III B is computed as follows:

Expressing the conditional probability of suppression $P(S/\theta,r)$ as a quotient of the number of successes S_i and the number of trials n_i leads to:

$$f_i = \frac{S_i}{n_i}$$

where the expectation of f is:

$$E[f_i] = P_i(S/\theta,r)$$

$$\ell n\ f_i = \ell n\left(\frac{S_i}{n_i}\right)$$

$$\mathrm{Var}[\ell n\ f_i] = \mathrm{Var}\left[\ell n\left(\frac{S_i}{n_i}\right)\right]$$

Since the random number S is binomial with mean

$$E(S) = nP(S/\theta,r)$$

and variance

$$\mathrm{Var}(S) = nP(S/\theta,r)\cdot(1 - P(S/\theta,r))$$

it can be stated as a function of a random variable U with mean

$$E(U) = 0$$

and variance

$$\mathrm{Var}(U) = 1$$

i.e.

$$S_i = n_i \cdot P_i(S/\theta,r) + \sqrt{n_i P_i(S/\theta,r)\cdot[1 - P_i(S/\theta,r)]}\cdot U$$

Inserting this in the equation above:

$$\mathrm{Var}[\ell n\ f_i] = \mathrm{Var}\left[\ell n\ \frac{n_i P_i(S/\theta,r) + \sqrt{n_i P_i(S/\theta,r)[1 - P_i(S/\theta,r)]}\cdot U}{n_i}\right]$$

$$\mathrm{Var}[\ell n\ f_i] = \mathrm{Var}\left[\ell n\left(P_i(s/\theta,r)\cdot\left(1 + \sqrt{\frac{[1 - P_i(S/\theta,r)]}{n_i P_i(S/\theta,r)}}\cdot U\right)\right)\right]$$

$$\mathrm{Var}[\ell n\ f_i] = \mathrm{Var}\left[\ell n\ P_i(S/\theta,r) + \ell n\left(1 + \frac{[1 - P_i(S/\theta,r)]}{n_i P_i(S/\theta,r)}\cdot U\right)\right]$$

Since $P(S/\theta,r)$ is constant we know that

$$\mathrm{Var}(\ell n\ P(S/\theta,r)) = 0$$

which leads to:

$$\mathrm{Var}[\ell n\ f_i] = \mathrm{Var}\left[\ell n\left(1 + \sqrt{\frac{1 - P_i(S/\theta,r)}{n_i P_i(S/\theta,r)}}\cdot U\right)\right]$$

knowing that for small x's

$$\ell n(1 + x) \sim x \quad \text{for} \quad x <<$$

we can state

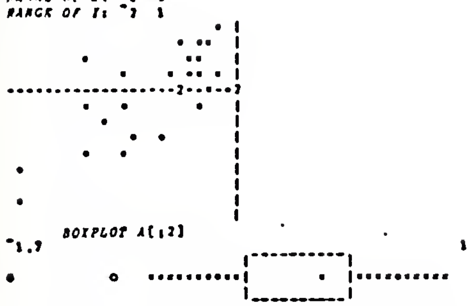
$$\mathrm{Var}[\ell n\ f_i] \approx \mathrm{Var}\left[\sqrt{\frac{1 - P_i(S/\theta,r)}{n_i\ P_i(S/\theta,r)}}\cdot U\right]$$

APPENDIX D

ANALYSIS OF RESIDUALS 81 mm

Method 1

```
      REGRES
      BETA
¯0.0002098040818   7.776799378E¯1
      R
4784.706205
      EPS
0.7514534578
      SCAT A
RANGE OF X: ¯2  0
RANGE OF Y: ¯2  1
```

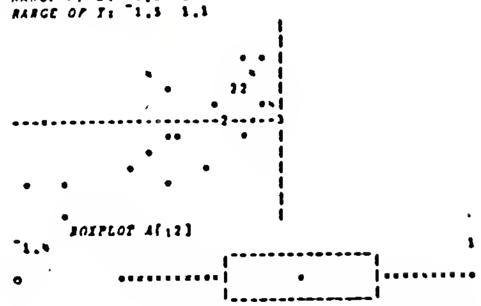


• RESIDUALS

0.1225134611	¯0.7033744874
0.07756195129	¯0.9275033515
0.6093274828	¯0.5736432766
0.1225134611	0.1762237584
0.7793328131	0.7872211385
0.7315574685	0.211873729
0.06013805738	0.1585230213
0.118790163	¯0.3549210564
0.2816405578	¯0.3586651295
0.1011406093	¯0.6473528446
0.4812845054	¯1.728110947
0.9738767177	¯1.131855010
¯0.1983759208	0.8733127153
¯0.06000278378	0.3875880509
¯0.3122341829	

Method 2

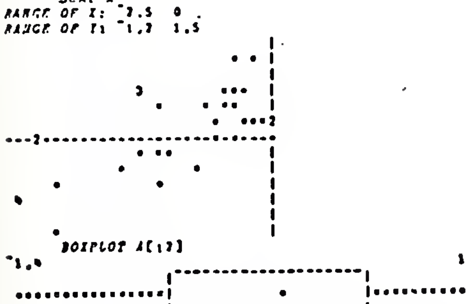
```
      REGRES
      BETA
¯0.0004532133908   0.0007746645695
      R
2215.733949
      EPS
0.7180716485
      SCAT A
RANGE OF X: ¯3.5  5
RANGE OF Y: ¯1.5  1.1
```



¯0.1147720006	¯0.6391549172
0.07346092129	¯0.8316365432
0.5382956437	¯0.534509278
0.1147224406	0.5702334871
0.2481587311	1.040309837
0.6614387041	0.4297728957
0.05284970369	0.55753391
0.08787606107	¯0.6712654436
0.2115214135	¯0.09951621467
0.1659101775	¯0.2533419559
0.4991445546	¯1.444455332
0.9630107202	¯0.913861551
¯0.1534063596	¯0.9385863989
¯0.07413597540	0.8478982533
¯0.3232001823	

Method 3

```
      REGRES
      BETA
¯0.0004041250149   0.0007759433421
      R
2474.491814
      EPS
0.8373271458
      SCAT A
RANGE OF X: ¯2.5  0
RANGE OF Y: ¯1.2  1.5
```



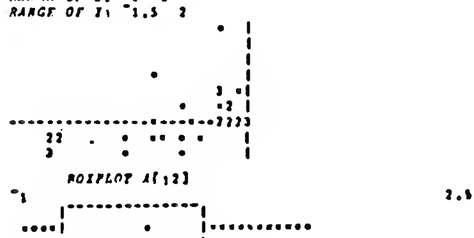
• RESIDUALS

0.1033415955	0.3444645073
0.1491747247	1.149411417
0.7059752278	0.3840758368
0.1333615055	0.5207463454
0.3227127706	¯0.6164447331
0.829707868	0.0318651 2145
0.07144410177	¯0.2775283905
0.1621790306	¯1.383614103
0.3747707973	¯2.257438 9159
0.1305332097	¯0.8023527737
0.6342867637	0.7400211342
1.113516316	0.031485972185
¯0.1149710200	0.0311415 72245
0.0041516667	¯0.8472527777
¯0.1546795855	¯0.0427547777
¯0.8745451587	¯0.8847527777
¯0.8171480091	0.7200211342
¯0.2660314801	0.7200231942

Method 4

```
      REGRESS
      BETA
¯0.0006281647221   0.0005116237212
      R
1591.938789
      EPS
0.5142910348

      SCAT A
RANGE OF X: ¯2  0
RANGE OF Y: ¯1.5  2
```



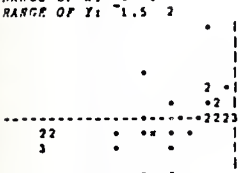
0.8317476736	0.1007071212
1.147447348	¯0.07377144582
0.5414905103	0.4410632278
0.8129742447	0.1947071712
¯0.0724017519	0.2148161675
0.03142721011	0.5246713732
0.1075417940	0.02844160 75
¯1.014711417	0.0511357811
¯0.5518127155	0.1354481459
2.755500618	0.3114184622
¯0.8476364447	0.5111717675
¯0.7054111807	0.5044751134
¯0.5744401704	¯0.4417740781
¯0.9730144958	¯0.002101114156
1.0210442707	¯0.2717414472
¯0.5944010704	¯0.5742164761
¯0.9190118451	¯0.7795711947
¯0.705811489	¯0.5814723378

APPENDIX D

ANALYSIS OF THE RESIDUALS 81 mm

Method 5 1st Iteration

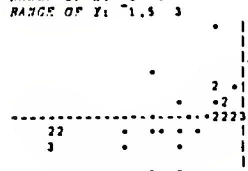
```
ITER P
BETIT
¯0.0006237481418  0.0005173232419
K
1577.913897
EPS
0.8182915217
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯1.5  2
```



BOXPLOT A[;2]

After 8 Iterations

```
ITER P
BETIT
¯0.0008335983121  0.0005171736813
K
1578.287323
EPS
0.818248515
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯1.5  3
```

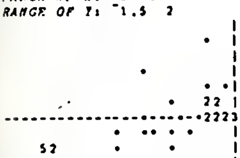


BOXPLOT A[;2]

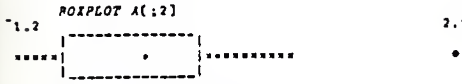
```
 0.8413525808     0.1047821677
 1.190158517     ¯0.02275444607
 0.5491747565     0.4827330553
 0.8411021493     0.1047921677
¯0.4714170799     0.21513043
 0.0342514563     0.5753668109
 0.1171660997     0.02992040711
¯1.000725545      3.0514559725
¯0.5460580008     0.1361635632
 2.700701546      0.21327.5355
¯0.8295405817     0.5136439616
¯0.6879065601     0.9906222356
¯0.5691142541    ¯0.08149033455
¯0.958967653     ¯0.000313067125783
 1.494688236     ¯0.2755550401
¯0.5691142541    ¯0.5923629403
¯0.958989659     ¯0.7881924452
¯0.6879865601    ¯0.5871602558
```

Method 6 1st Iteration

```
ITER P
BETIT
¯0.000586924353  0.0004745912294
K
1703.797082
EPS
0.8006073221
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯1.5  2
```

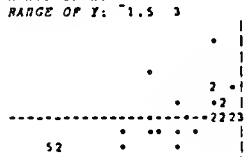


BOXPLOT A[;2]

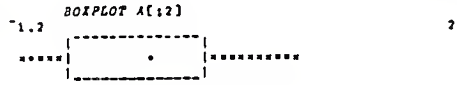


After 8 Iterations

```
ITER P
BETIT
¯0.0005879905689  0.0004755868021
K
1700.70755
EPS
0.808833725
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯1.5  3
```



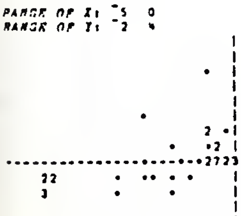
BOXPLOT A[;2]



```
 0.1011635701      0.751553814
¯0.04537068075     1.098461082
 0.4501712773      0.4494080825
 0.1011635701      0.7519523831
 0.2006621028     ¯0.5078504003
 0.542799433      ¯0.66651521771
 0.02540180956     0.02736793288
 0.03698158235    ¯1.093159865
 0.1035961852     ¯0.6458246028
 0.1929131492      2.553959744
 0.4664417717     ¯1.023260737
 0.8592470348     ¯0.880939228
¯0.1017997208     ¯0.7959343433
¯0.03126189117    ¯1.153999264
¯0.3229332499      1.301735568
¯0.6127223331     ¯0.7959343433
¯0.8185507038     ¯1.153999264
¯0.6346214566     ¯0.380939228
```

Method 7 1st Iteration

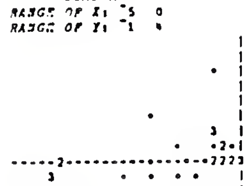
```
ITER P
BETIT
¯0.0006994308921  0.0006014240416
K
1429.733618
EPS
0.8598782914
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯2  4
```



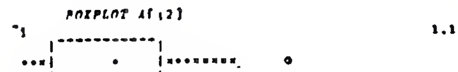
BOXPLOT A[;2]

After 8 Iterations

```
ITER P
BETIT
¯0.0007350245001  0.0006404882024
K
1360.498868
EPS
0.8712807531
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯1  4
```



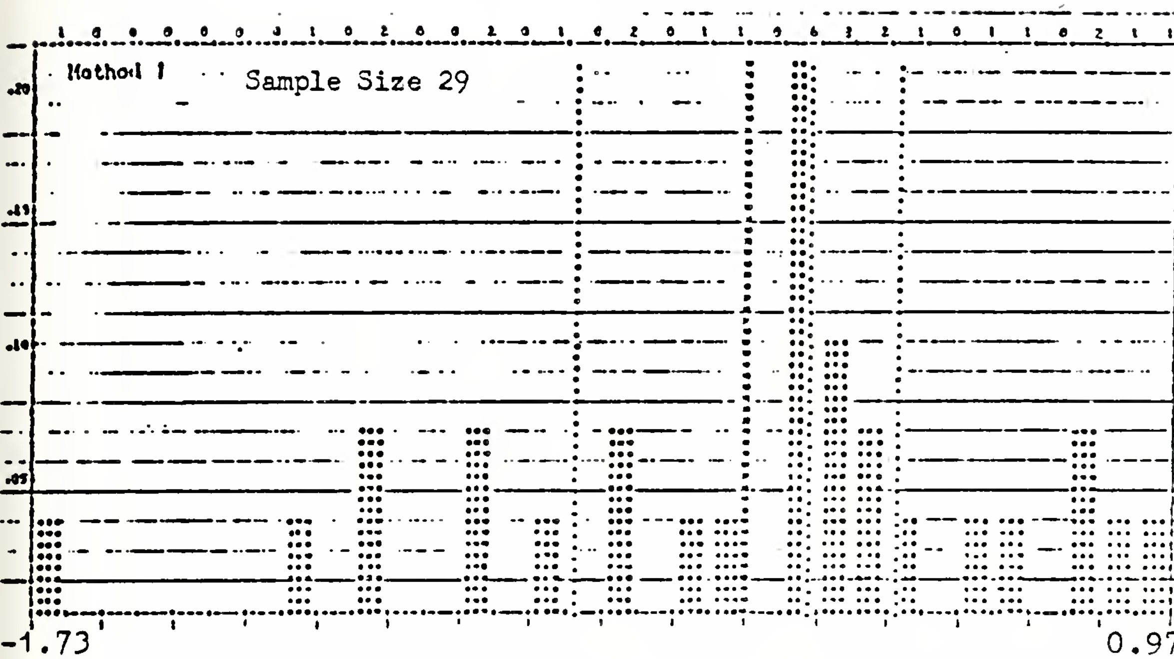
BOXPLOT A[;2]

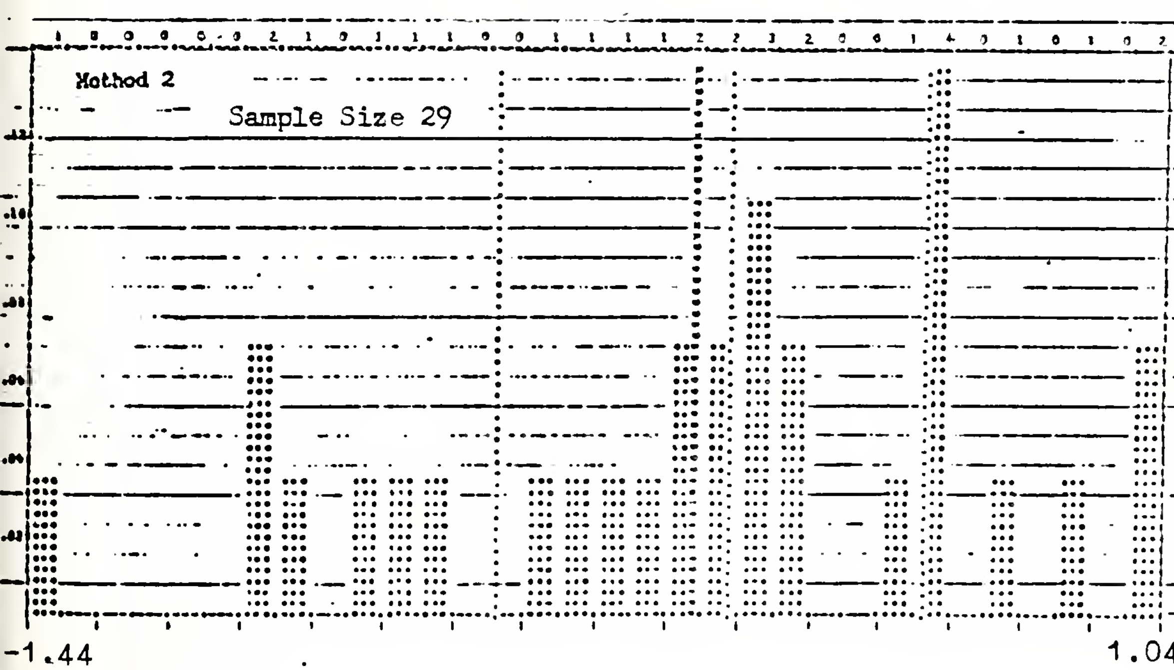


```
 0.9558553662      0.09508446787
 1.15087603       ¯0.1456650699
 0.450659763       0.7054599357
 0.9561833553      0.09500446787
¯0.4099327811      3.1363456938
¯0.065326153718    0.7980875127
 0.2316675051      0.009322707307
¯0.9947027463     ¯0.02733487668
¯0.6445730023     ¯0.04111573508
 3.148116327       0.2140916857
¯0.71413621        0.3856480910
¯0.6611436654      0.7422156319
¯0.2017871599     ¯0.08079918435
¯0.7750937167     ¯0.66729463319
 1.521511131      ¯0.4409714438
¯0.2017871599     ¯0.5916317966
¯0.7750937167     ¯0.4475838658
¯0.6611436654     ¯0.7516526595
```

APPENDIX D

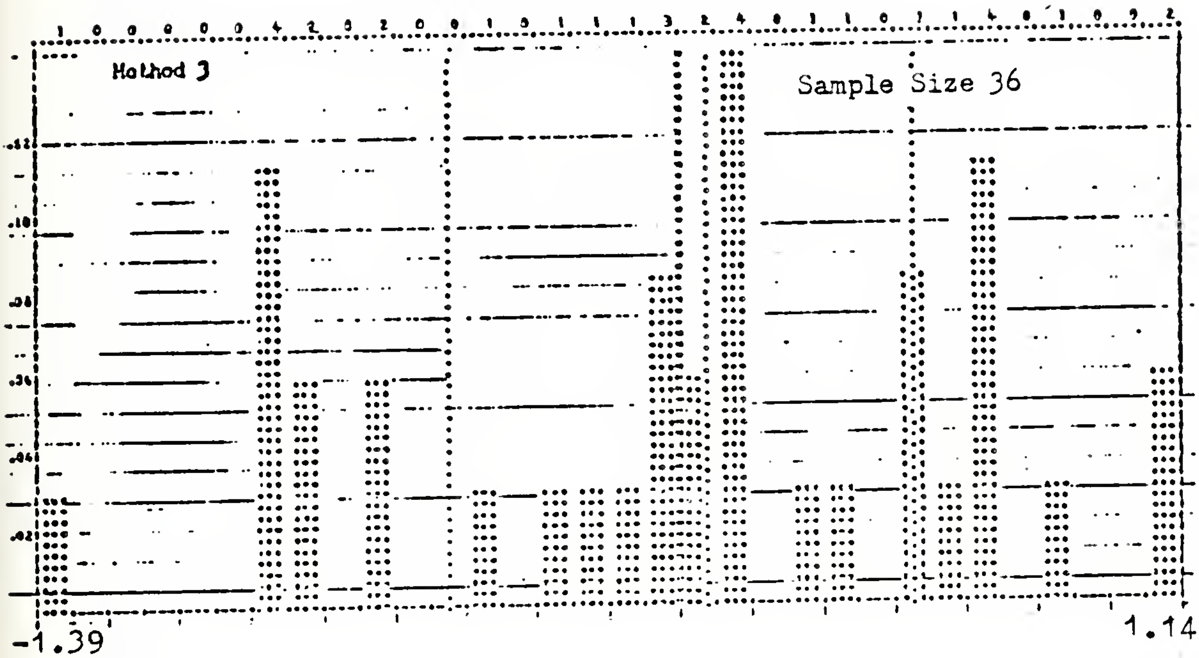
ANAYSIS OF THE RESIDUALS 81 mm

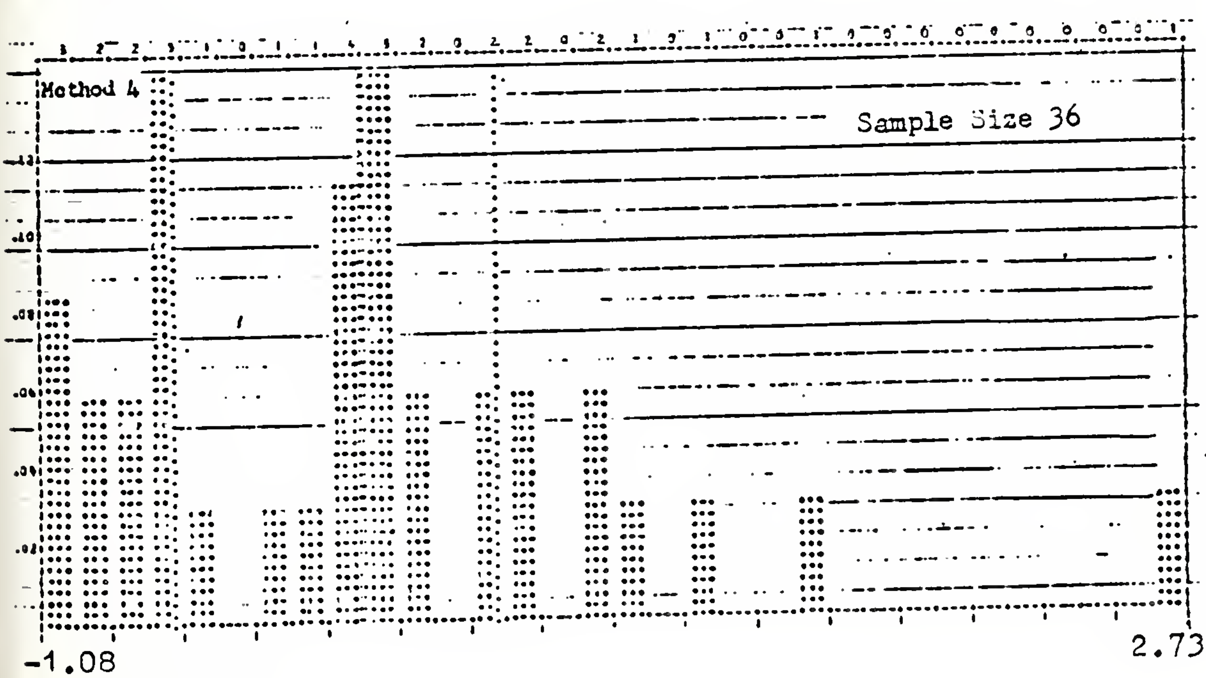




APPENDIX D

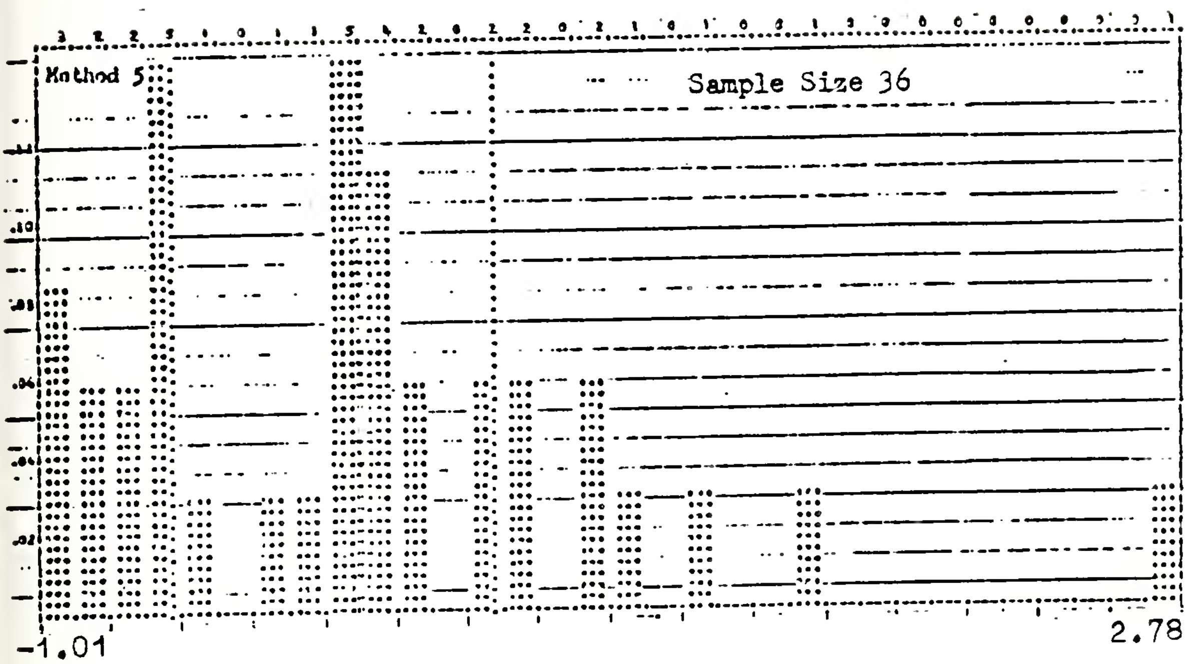
ANALYSIS OF THE RESIDUALS 81 mm

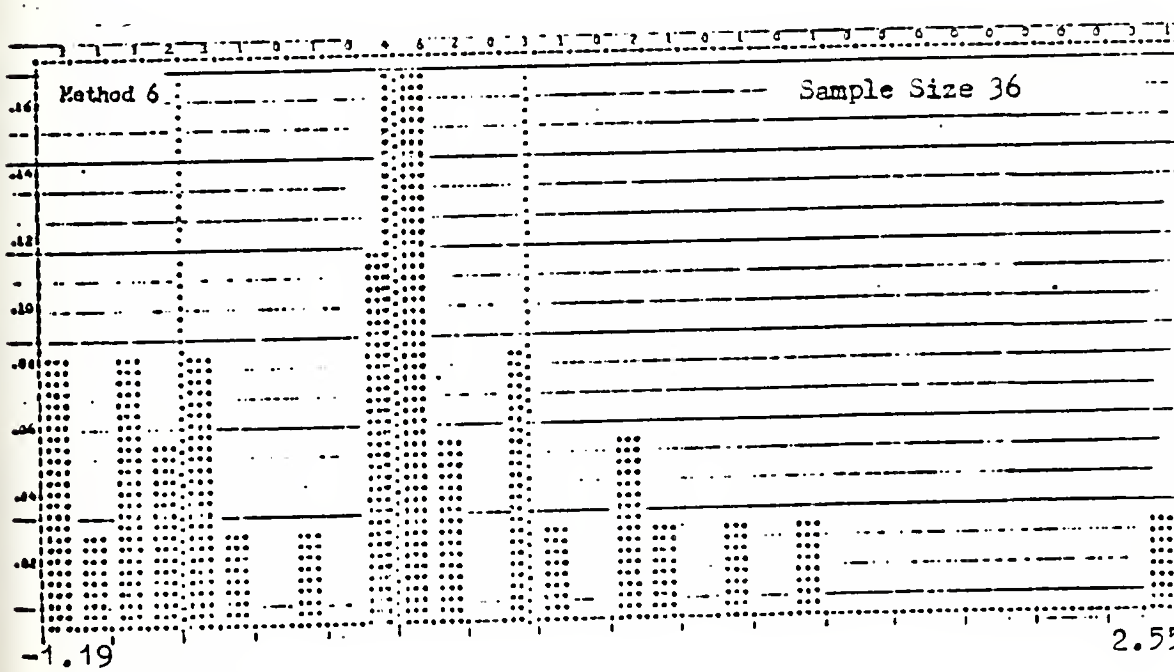




APPENDIX D

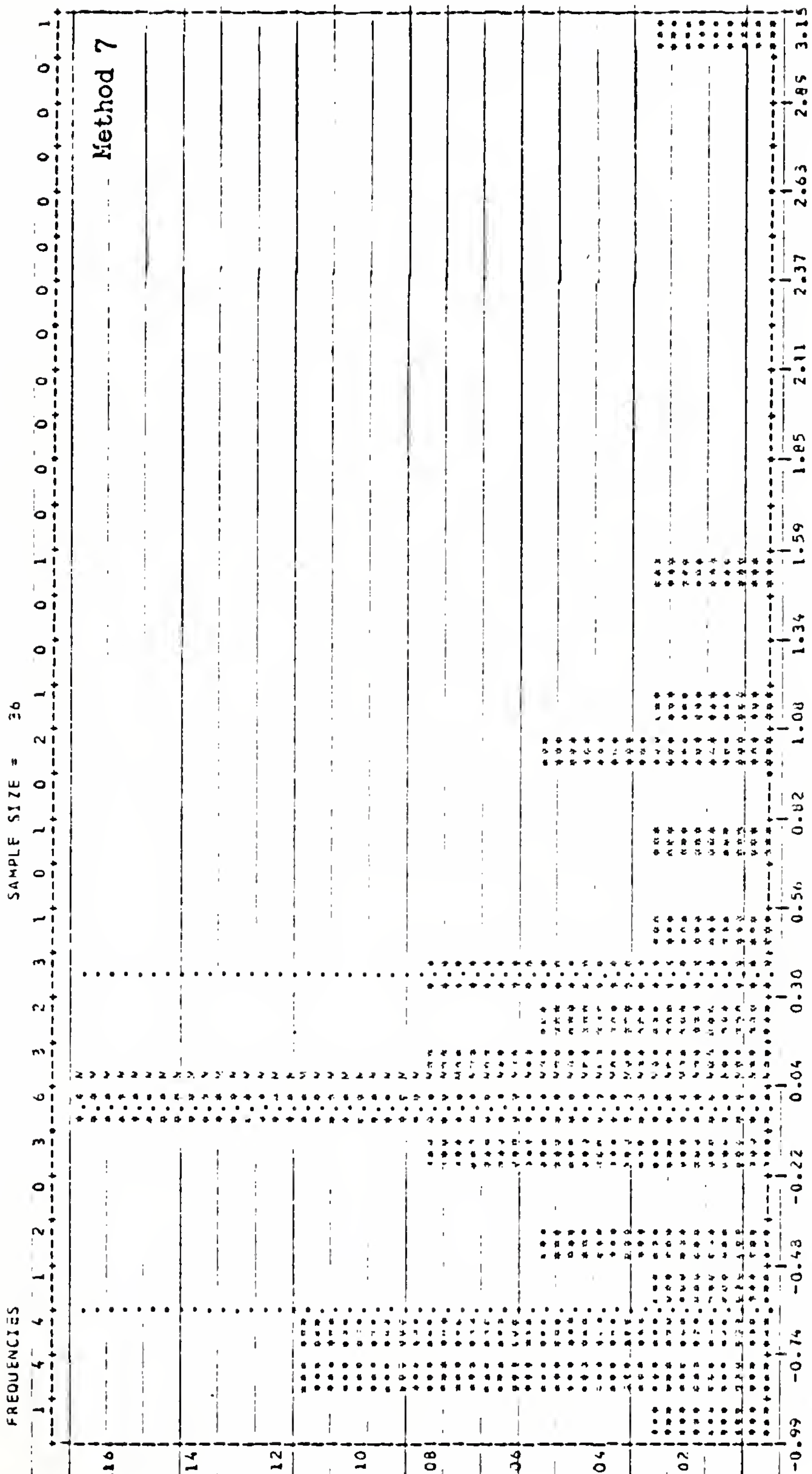
ANALYSIS OF THE RESIDUALS 81 mm





APPENDIX D

ANALYSIS OF THE RESIDUALS 81 mm

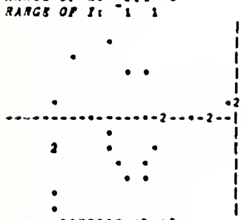


APPENDIX E

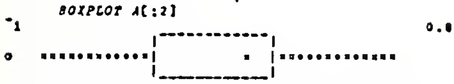
ANALYSIS OF THE RESIDUALS 105 mm

Method 1

```
      REGRES
      BETA
¯0.0033187639576   0.0002319729873
      K
3137.117532
      EPS
0.7277255255
      SCAT A
RANGE OF X: ¯2.5  0
RANGE OF X: ¯1  1
```



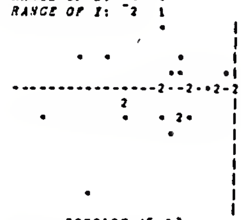
```
      BOXPLOT A[;2]
```



```
*RESIDUALS   ¯0.9919574599    0.436441781
             ¯0.5602283478    0.4451161506
             ¯0.0332327082    0.02393363272
             ¯0.7263790998    0.04672915095
              0.0053374078   ¯0.3639285719
             ¯0.03276200988  ¯0.4661890376
             ¯0.6845239743    0.1846141216
              0.6267373394    0.02476542064
             ¯0.2242573151    0.1063189386
             ¯0.8801937825    0.1063189386
              0.2434331785    0.8327340353
              0.3486534481   ¯0.3985771098
             ¯0.3700568868   ¯0.3385771098
```

Method 2

```
      REGRES
      BETA
¯0.0004510183734   0.0003713826479
      K
2217.204573
      EPS
0.8247615262
      SCAT A
RANGE OF X: ¯4  0
RANGE OF Y: ¯2  1
```

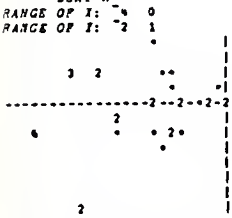


```
      BOXPLOT A[;2]
```

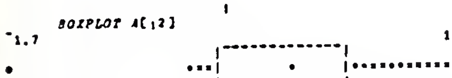
```
 0.3627351434    ¯1.702715717
 0.314352512     ¯0.5426013786
¯0.05361281613    0.005521829446
¯0.03062323791   ¯0.6870253511
¯0.4104620832     0.3922170832
¯0.5158226349     0.02692391943
 0.15669524      ¯0.624838045
¯0.003153460954   0.521412498
 0.05681876363    0.03933189525
 0.03681876363   ¯0.6145345772
 0.3425332303    ¯0.5107061304
¯0.2309866134     0.3756930127
¯0.2309866134    ¯0.04301732214
```

Method 3

```
      REGRES
      BETA
¯0.0004591993072   0.0003792447864
      K
2177.751059
      EPS
0.8259307351
      SCAT A
RANGE OF X: ¯4  0
RANGE OF Y: ¯2  1
```



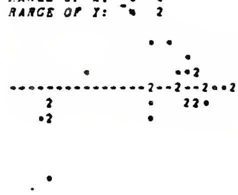
```
      BOXPLOT A[;2]
```



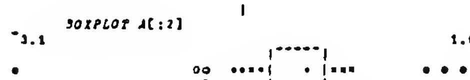
```
*RESIDUALS    0.3823032461    ¯0.6183680213
              0.3274111544     0.06764315426
             ¯0.04452486202   ¯0.5712833131
             ¯0.0215353438     0.3788780886
             ¯0.4046457786    ¯0.0178122463
             ¯0.5120061142    ¯1.683454043
              0.1575569035    ¯1.683454043
              0.0001187025236  0.4334373964
              0.03773231801    0.4334373394
              0.03773703801    0.5757795269
              0.9750904063     0.5757795249
             ¯0.2043469126     0.5757795269
             ¯0.2043469126    ¯0.4588017053
             ¯0.5322147451    ¯0.4588017053
              0.01341723002   ¯0.4588017053
             ¯0.6777228905    ¯0.4588017053
              0.03277334305
```

Method 4

```
      RECRES
      BETA
¯0.0004727356255   0.0004083957056
      K
2029.405871
      EPS
0.8300514203
      SCAT A
RARGE OF X: ¯5  0
RARGE OF Y: ¯4  2
```



```
      BOXPLOT A[;2]
```



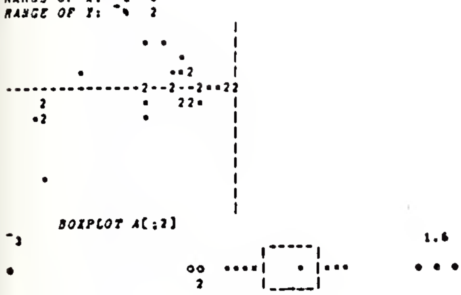
```
 0.05733773479    0.533105519
¯0.6075178139     0.4476172367
 0.5747121385     0.01332775476
 0.5338668373     0.03610785783
 0.0615225563    ¯0.3281707726
¯0.5760253097    ¯0.4241571484
¯0.3567253355     0.1757580345
¯0.3567253355     0.01897573539
 1.620281206      0.1025811566
 0.2042290371     0.1025811566
¯0.3360542829     1.173311515
¯1.071260288      0.005222973046
¯1.073260288      0.005229973046
 1.414787651     ¯0.8417437
¯1.116523377     ¯3.059857836
 0.5122728737    ¯0.37436369
 0.1083860774
```

APPENDIX E

ANALYSIS OF THE RESIDUALS 105 mm

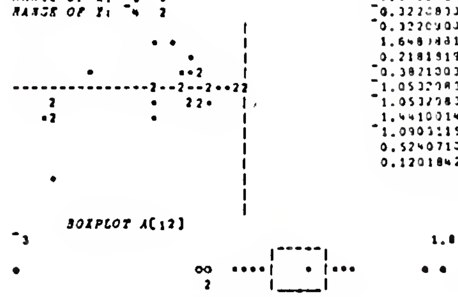
Method 5 — 1st Iteration

```
ITER P
BETIT
¯0.0004963385832  0.0004113757003
K
2014.753706
EPS
0.8288707169
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯4  2
```



After 8 Iterations

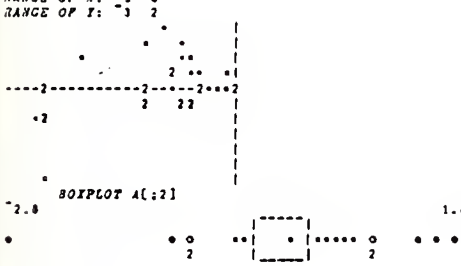
```
ITER P
BETIT
¯0.0034963023523  0.0004113938476
K
2014.930788
EPS
0.828317787
SCAT A
RANGE OF X: ¯3  0
RANGE OF Y: ¯4  2
```



¯0.6030131435	0.5548673078
0.6153223553	0.4873697043
0.5534517465	0.0256155054
0.0652190617	0.04773571346
¯0.5722487564	¯0.3796935482
¯0.3222833384	¯0.416678474
¯0.3220903394	0.1733656608
1.648388127	0.02318230158
0.2181919307	0.1043127183
¯0.3821303312	0.1040127133
¯1.053278384	1.277902747
¯1.053278384	0.02591777328
1.441001437	0.02591777328
¯1.090311584	¯0.827164749
0.5240713379	¯3.345064771
0.1201842356	¯3.3372576688
	0.06384440527

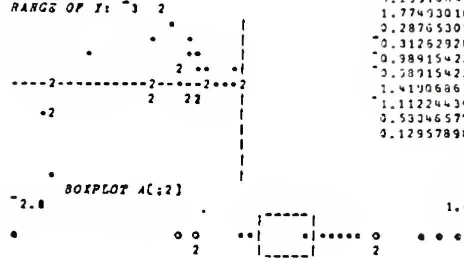
Method 6 — 1st Iteration

```
ITER P
BETIT
¯0.000482337397  0.000376259559
K
2073.237543
EPS
0.780336177
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```



After 8 Iterations

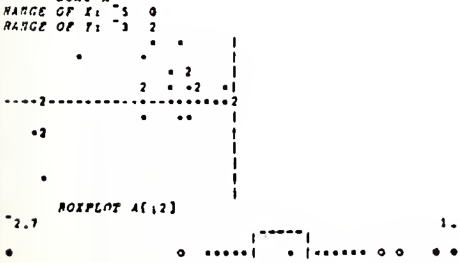
```
ITER P
BETIT
¯0.0004830592895  0.0003778769877
K
2070.13926
EPS
0.7822579875
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```



¯0.529615444	0.3782950349
0.9616371006	0.6093953746
0.7175005156	0.223357576
0.03978175541	0.2505336731
¯0.5475661011	¯0.120335747
¯0.1371064249	¯0.2559206228
¯0.1331064249	0.2529511312
1.774930167	0.09616903101
0.2876530559	0.1286483197
¯0.312629266	0.1286483157
¯0.9891542297	1.573164767
¯0.3891542297	0.2379123927
1.419068678	0.2371123427
¯1.112244345	¯3.6245277733
0.5334657724	¯2.34704371
0.1295789891	¯3.7584125327
	0.1372421547

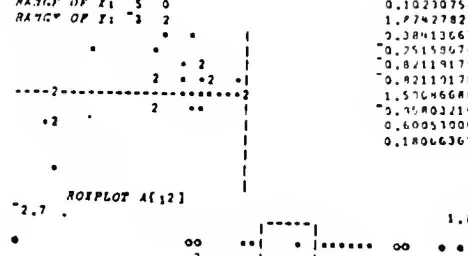
Method 7 — 1st Iteration

```
ITER P
BETIT
¯0.0005114408483  0.0003362984418
K
1955.260328
EPS
0.7748666206
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```



After 8 Iterations

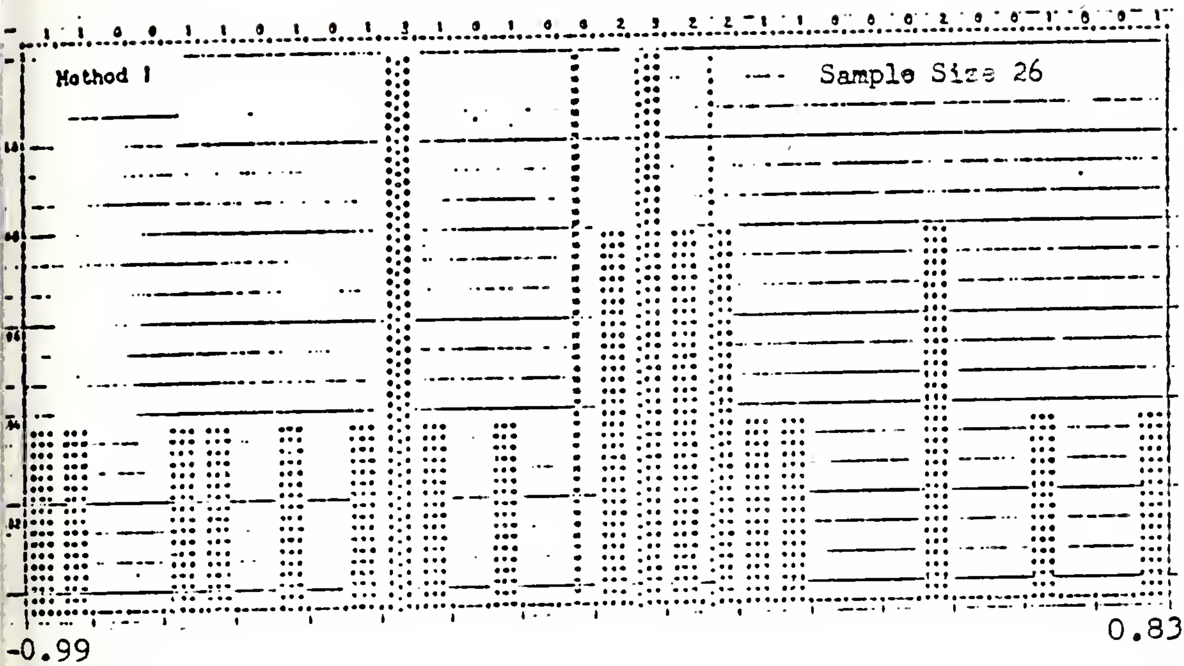
```
ITER P
BETIT
¯0.0005022220242  0.000386394017
K
1991.151228
EPS
0.7693689215
SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```

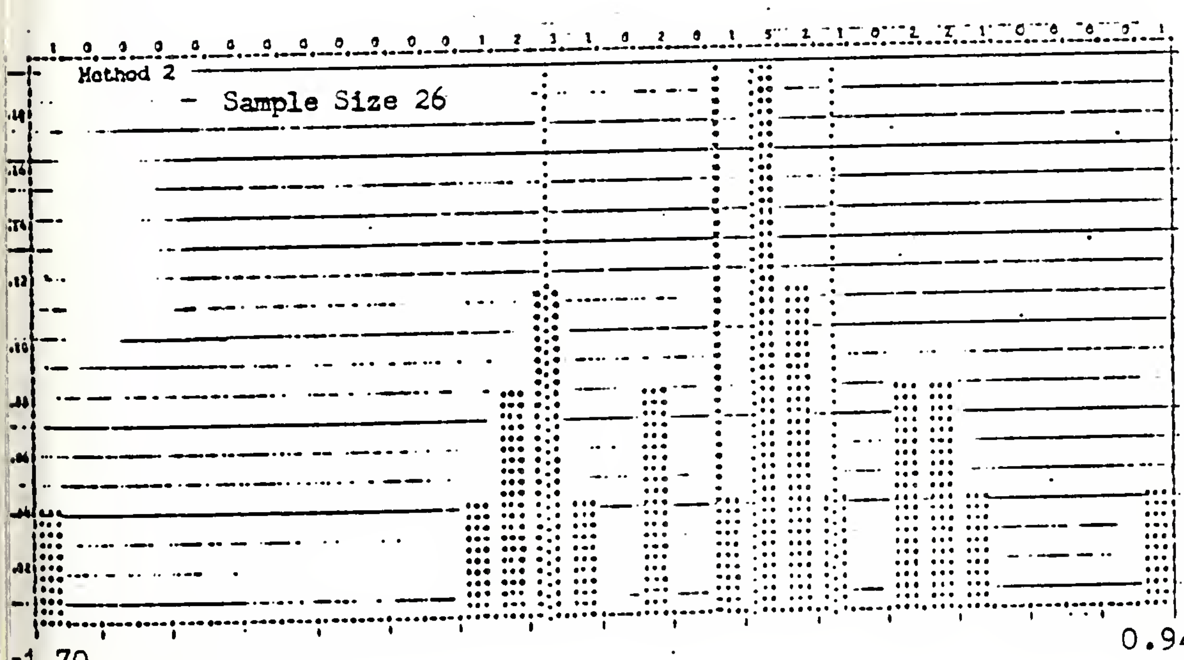


0.2126770601	¯1.128568109
¯0.506842957	1.018623806
1.268005126	0.3517542038
0.7276700013	0.351404175
0.1196137833	¯0.05720845086
¯0.4933077564	¯0.2262847809
0.132307555	0.2255837074
0.102307555	0.1144935619
1.874278221	0.1418933087
0.3841366784	0.1418833087
¯0.7515967836	1.813181673
¯0.8211917866	0.3649531116
¯0.9211917866	0.3648531116
1.570466881	¯0.5437977517
¯0.3580321411	¯2.723474548
0.6005300639	¯0.1253011908
0.1806636546	

APPENDIX E

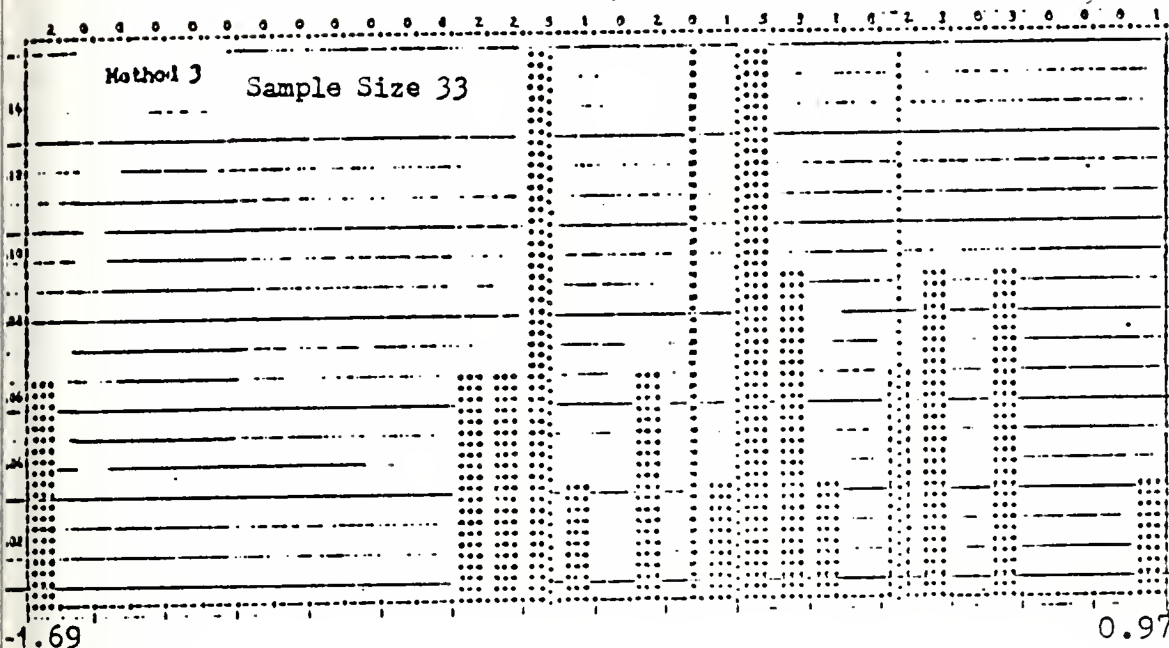
ANALYSIS OF THE RESIDUALS 105 mm

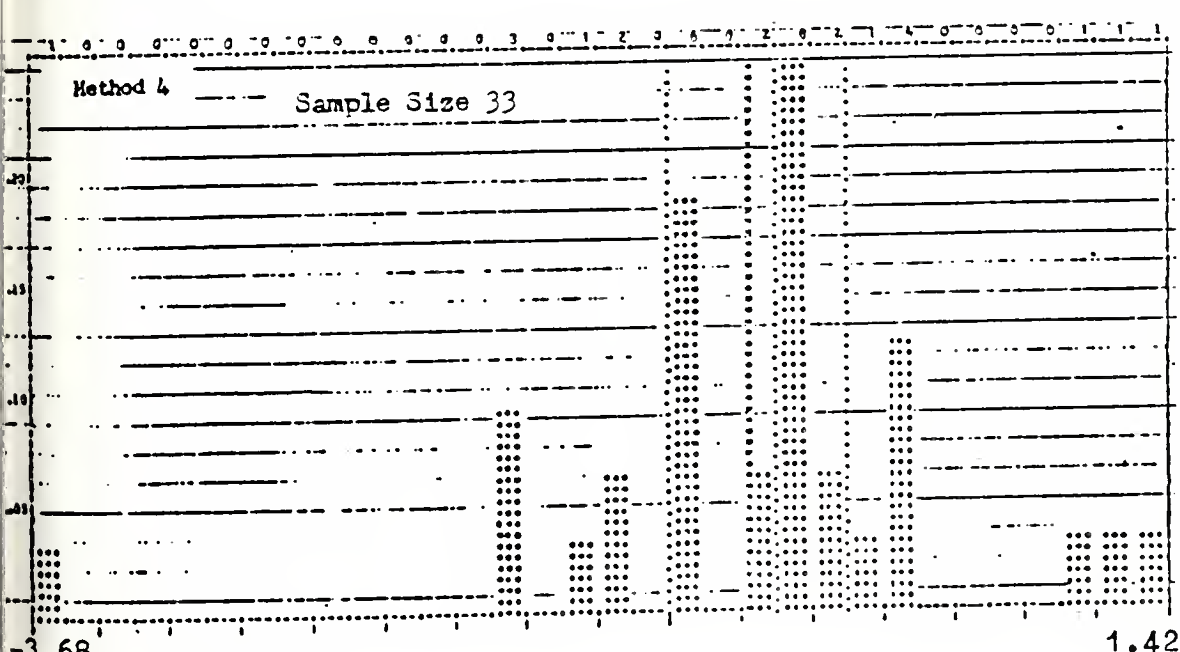




APPENDIX E

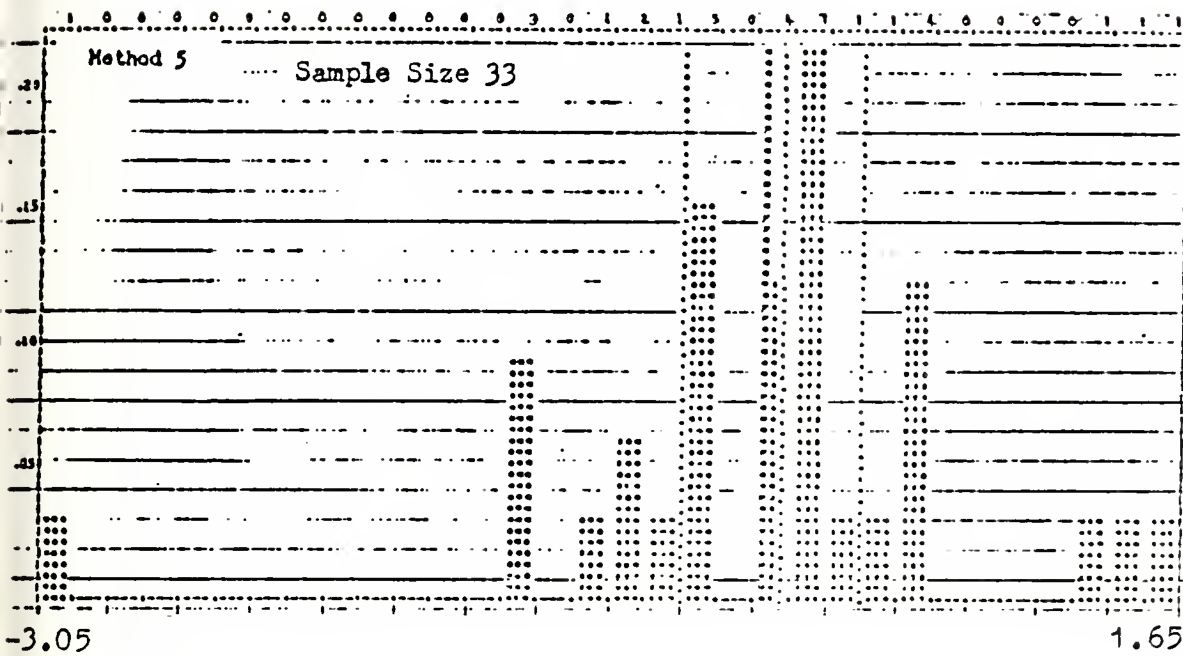
ANALYSIS OF THE RESIDUALS 105 mm

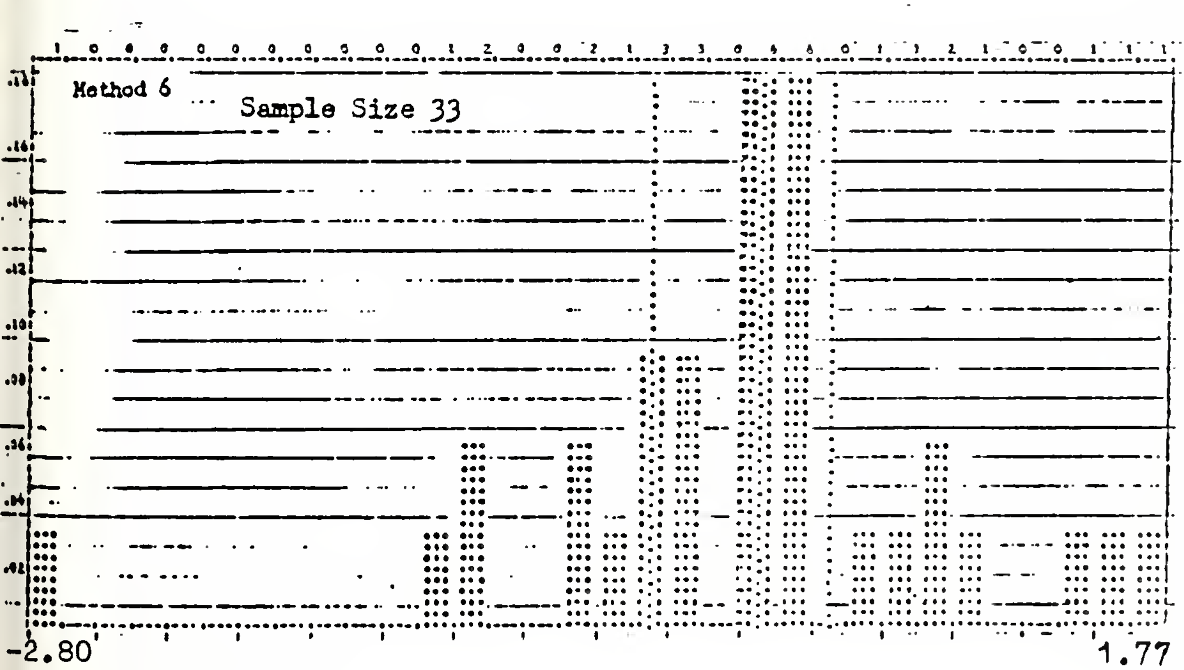




APPENDIX E

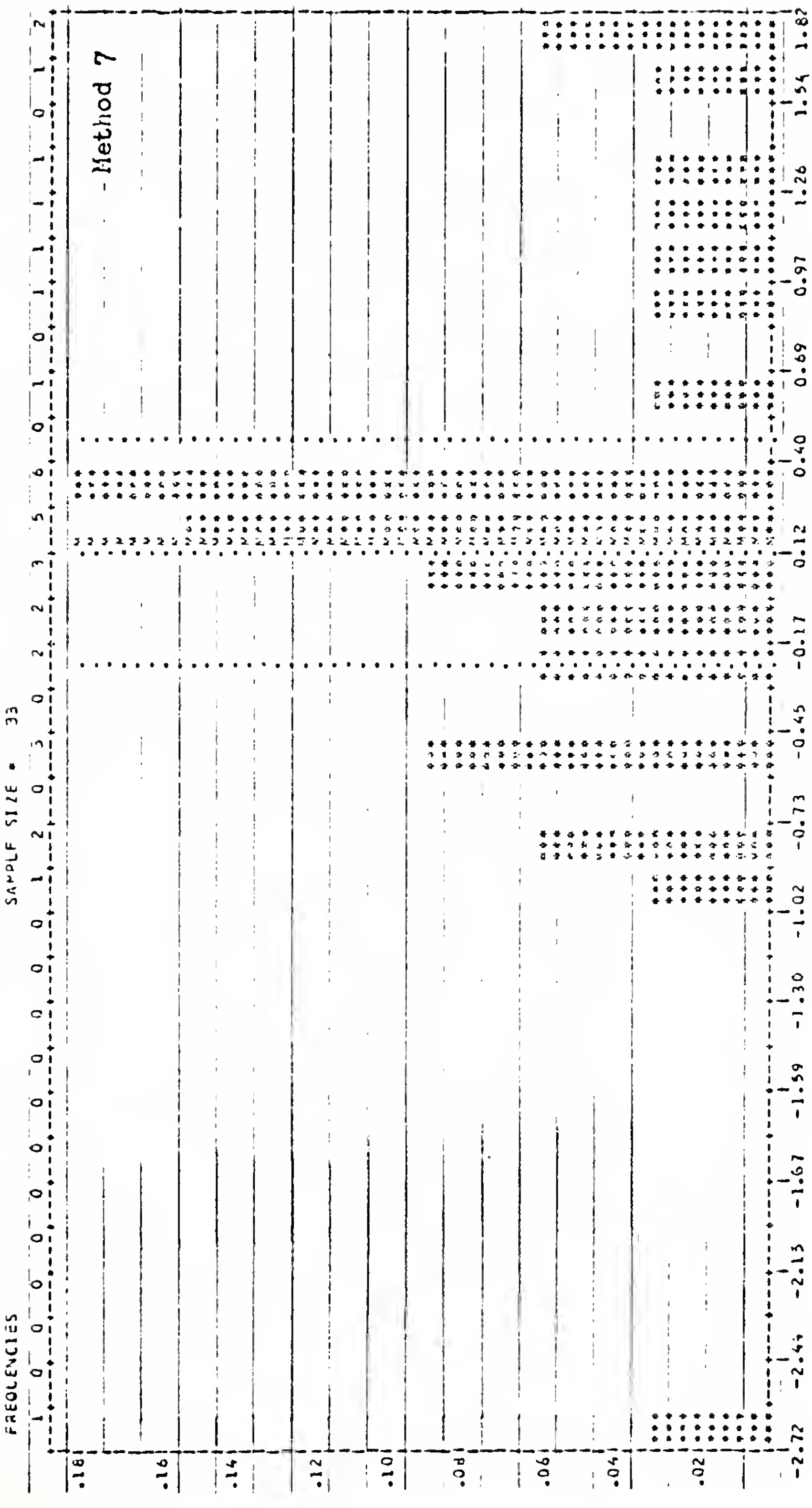
ANALYSIS OF THE RESIDUALS 105 mm





APPENDIX E

ANALYSIS OF THE RESIDUALS 105 mm



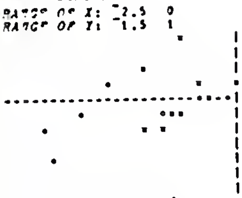
APPENDIX F

ANALYSIS OF THE RESIDUALS 155 mm

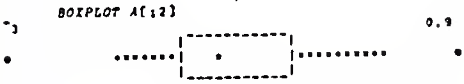
Method 1

```
      REGRES
      BETA
¯0.0003130588025  0.000283460777
      K
3126.37949
      EPS
0.3862053595

      SCAT A
RANGE OF X: ¯2.5  0
RANGE OF Y: ¯1.5  1
```



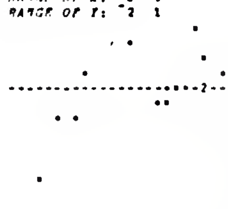
```
      BOXPLOT A[;2]
¯3                                              0.9
```



```
  RESIDUALS
 0.1057706323
¯0.1374023863
 0.1783503251
¯0.3805644431
¯0.2046101855
¯0.1101083006
¯0.4140541886
 0.6010147039
¯0.05212164292
¯0.1336173429
 0.185104369
 0.3240727187
¯1.04530377
¯0.1258441519
 0.9005641602
¯0.6167584543
```

Method 2

```
      REGRES
      BETA
¯0.00020532813086  0.0002468152474
      K
3346.064073
      EPS
0.8357322419
      SCAT A
RANGE OF X: ¯3  0
RANGE OF Y: ¯2  1
```

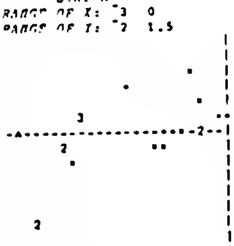


```
      BOXPLOT A[;2]
¯1.6                                            0.9
```

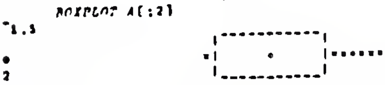
```
 0.1651424742
 0.007187488869
 0.2377131671
¯0.2330745681
¯0.413362738
¯0.05049303815
¯0.267016496S
 0.2636143579
 0.0074942105O5
 0.007420349217
 0.6682937275
 0.3765737531
¯1.504674014
¯0.07334311176
 0.3300674261
¯0.587255197
```

Method 3

```
      REGRES
      BETA
¯0.0003372475178  0.0002885794276
      K
2965.181201
      EPS
0.8556902939
      SCAT A
RANGE OF X: ¯3  0
RANGE OF Y: ¯2  1.5
```



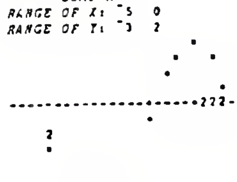
```
      BOXPLOT A[;2]
¯1.5                                            1.1
```



```
  RESIDUALS
 0.165302949      0.02019720043
 0.007065335697   0.02127300191
 0.2184736418     0.6873247075
¯0.2320766611     0.4349163238
¯0.3558614865    ¯1.540173255
¯0.3558614065    ¯1.540173255
¯0.01774337722   ¯0.0150008468
¯0.2531347638     1.077806814
 0.3153705112    ¯0.4374250098
 0.3153705112
 0.3153705112
```

Method 4

```
      REGRES
      BETA
¯0.0003742312562  0.0003099355847
      K
2672.144519
      EPS
0.828132674
      SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```



```
      BOXPLOT A[;2]
¯2.8                                            1.8
```

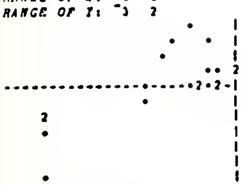
```
 0.2516762556
 0.2147616136
¯1.587056225
 0.3150487901
¯0.02887832137
 0.9631673905
 0.0456028036
¯0.03171763733
 1.7624625
 0.1350327041
 0.2459633308
 1.377301678
 0.5397405234
¯0.5877418472
¯0.9482386005
 0.1605377757
¯2.759288412
¯0.9482386005
 1.3541826
¯0.05797163555
```

APPENDIX F

ANALYSIS OF THE RESIDUALS 155 mm

Method 5 — 1st Iteration

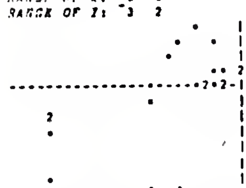
ITER P
BETIT
¯0.000381416803 0.0003184898682
K
2621.803738
EPS
0.8297213593
SCAT A
RANGE OF X: ¯5 0
RANGE OF Y: ¯3 2



BOXPLOT A[;2]

After 8 Iterations

ITER P
BETIT
¯0.000381:808985 0.0003162566401
K
2423.426315
EPS
0.829675092
SCAT A
RANGE OF X: ¯5 0
RANGE OF Y: ¯3 2



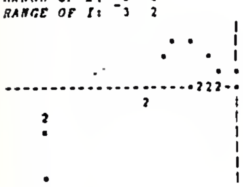
BOXPLOT A[;2]

0.2547563317
0.2773675158
¯1.561312747
0.318120R662
¯0.0212974191 6
0.9403:33088
0.0510204 6882
¯0.0:175 13214
1.790033931
0.1404 C0693
0.2559275872
1.404873069
0.613692236
¯0.5700667194
¯0.3125796459
0.1744883882
¯2.751615484
¯0.9:25796459
1.384420629
¯0.02773364634

Method 6 — 1st Iteration

ITER P
BETIT
¯0.000268154654 8 0.0002050498609
K
3729.191278
EPS
0.764670153

SCAT A
RANGE OF X: ¯5 0
RANGE OF Y: ¯3 2

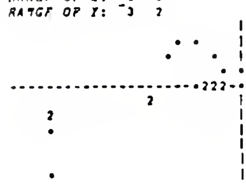


BOXPLOT A[;2]

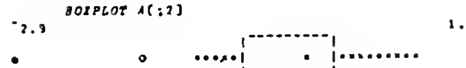
After 8 Iterations

ITER P
BETIT
¯0.0002699562953 0.0002066715375
K
3704.303316
EPS
0.7655740819

SCAT A
RANGE OF X: ¯5 0
RANGE OF Y: ¯3 2



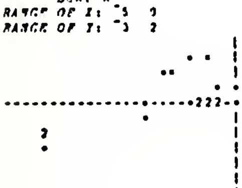
BOXPLOT A[;2]



0.2467277787
0.2025295582
¯1.627492772
0.3100053132
¯0.04113037668
0.922733344
0.01057127132
¯0.07418101907
1.689205787
0.1000011729
0.2034501495
1.304644929
0.4501778876
¯0.7318029508
¯1.1263827
0.01097463983
¯2.913349615
¯1.1263827
0.9805162197
¯0.4316380559

Mehtod 7 — 1st Iteration

ITER P
BETIT
¯0.0003948485558 0.0003339896846
K
2532.616582
EPS
0.8458677628
SCAT A
RANGE OF X: ¯5 0
RANGE OF Y: ¯3 2

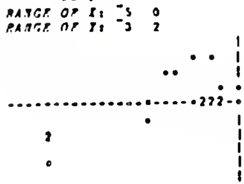


BOXPLOT A[;2]

After 8 Iterations

ITER P
BETIT
¯0.0004255598866 0.0003659148 33
K
2349.845536
EPS
0.8598433369

SCAT A
RANGE OF X: ¯5 0
RANGE OF Y: ¯3 2

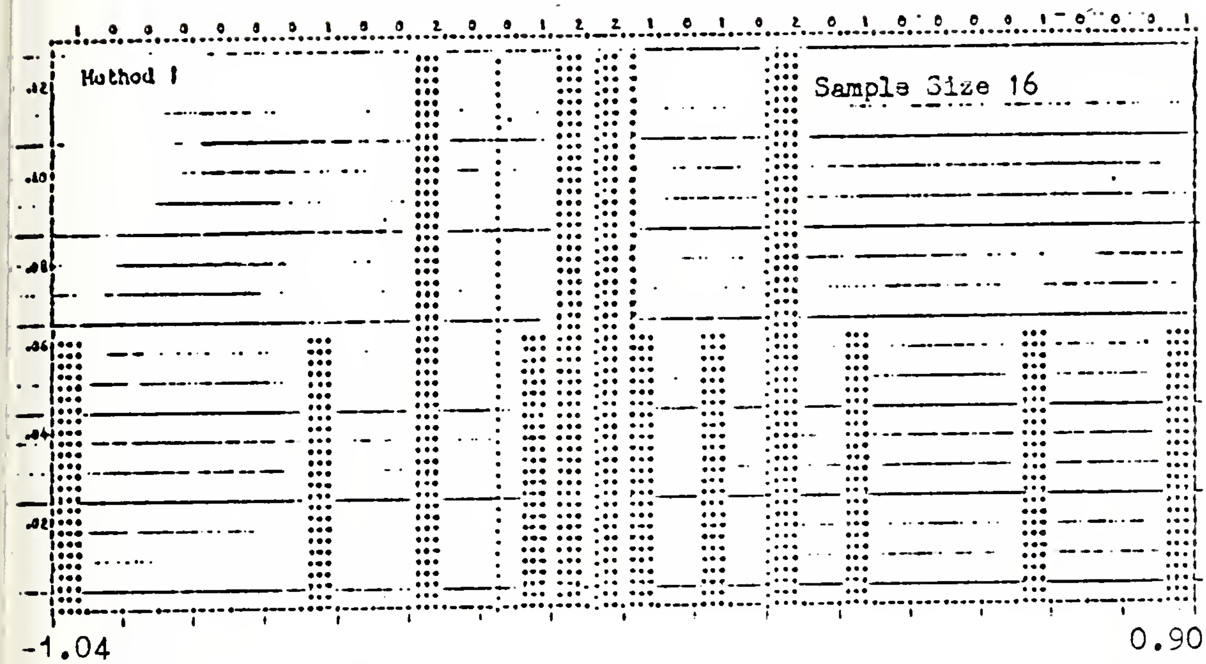


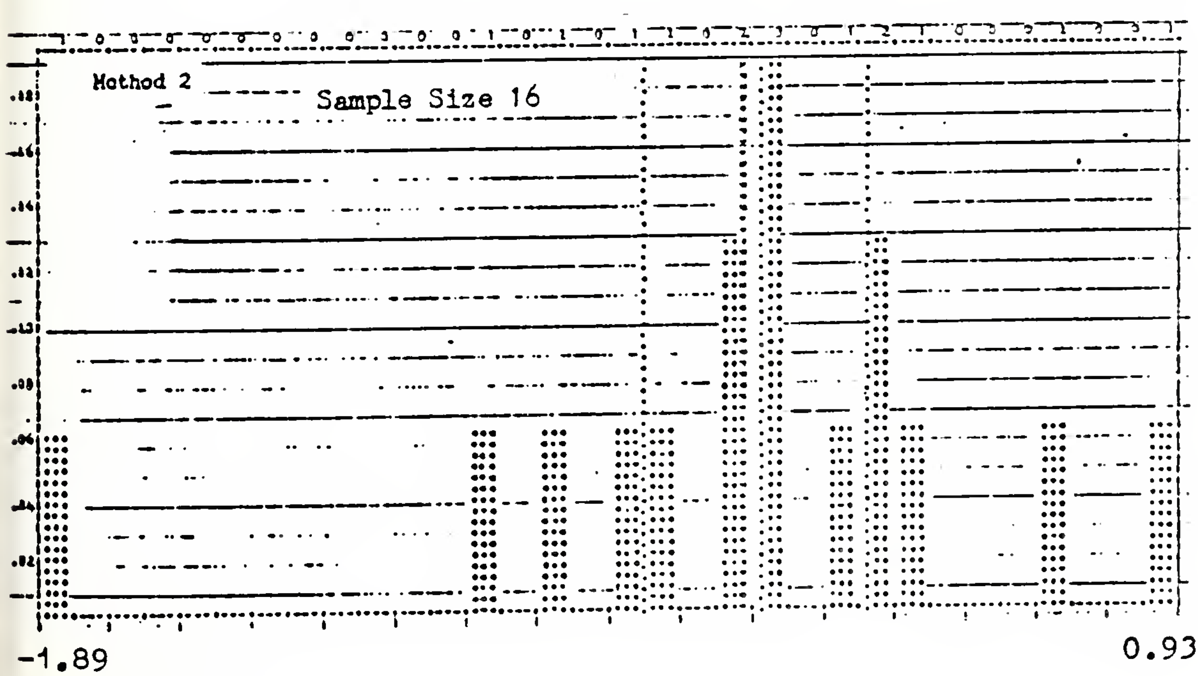
BOXPLOT A[;2]

0.2228882284
0.1564931382
¯1.773000337
0.7927697629
¯0.08517073676
0.7771451784
0.03456601217
¯0.07625730852
1.530724642
0.1239959177
0.2014716601
1.20456378
0.0378167275
¯0.5597257732
¯1.078748463
0.1976136798
¯2.772427038
¯1.078748463
1.499185088
0.07703081277

APPENDIX F

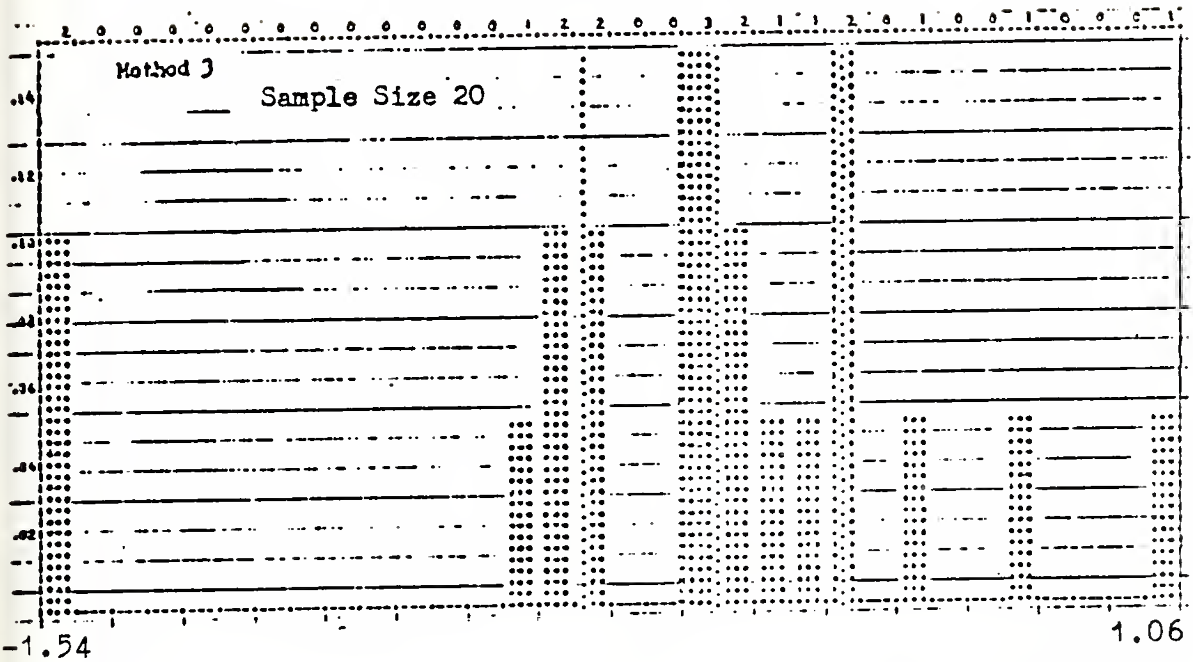
ANALYSIS OF THE RESIDUALS 155 mm

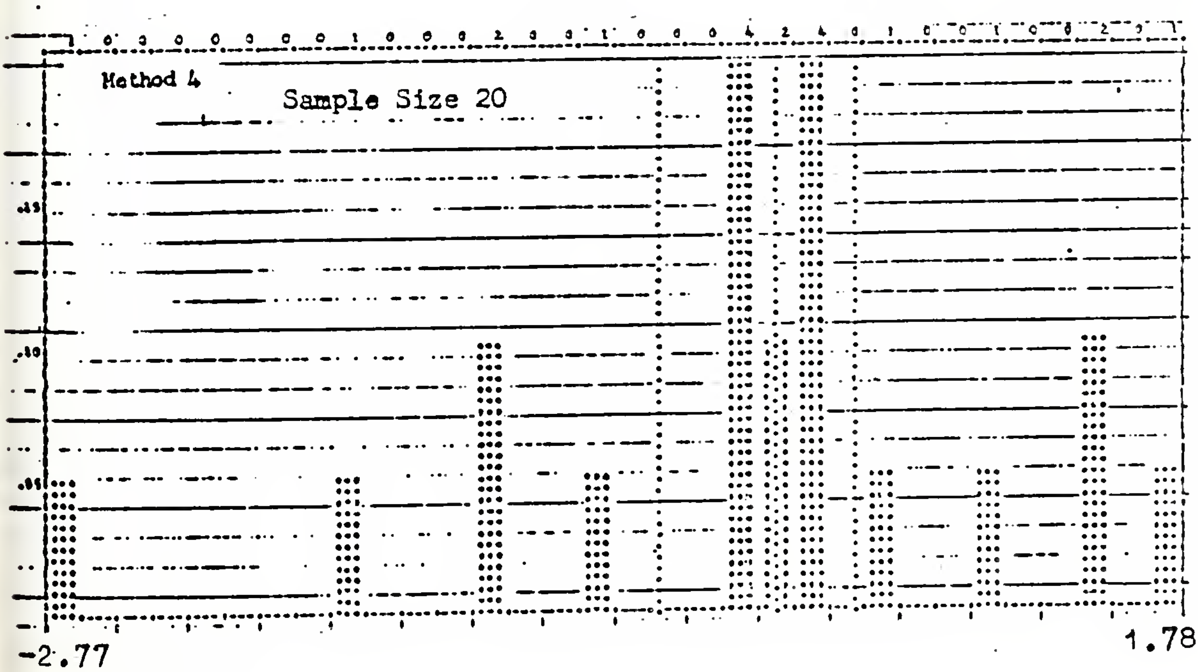




APPENDIX F

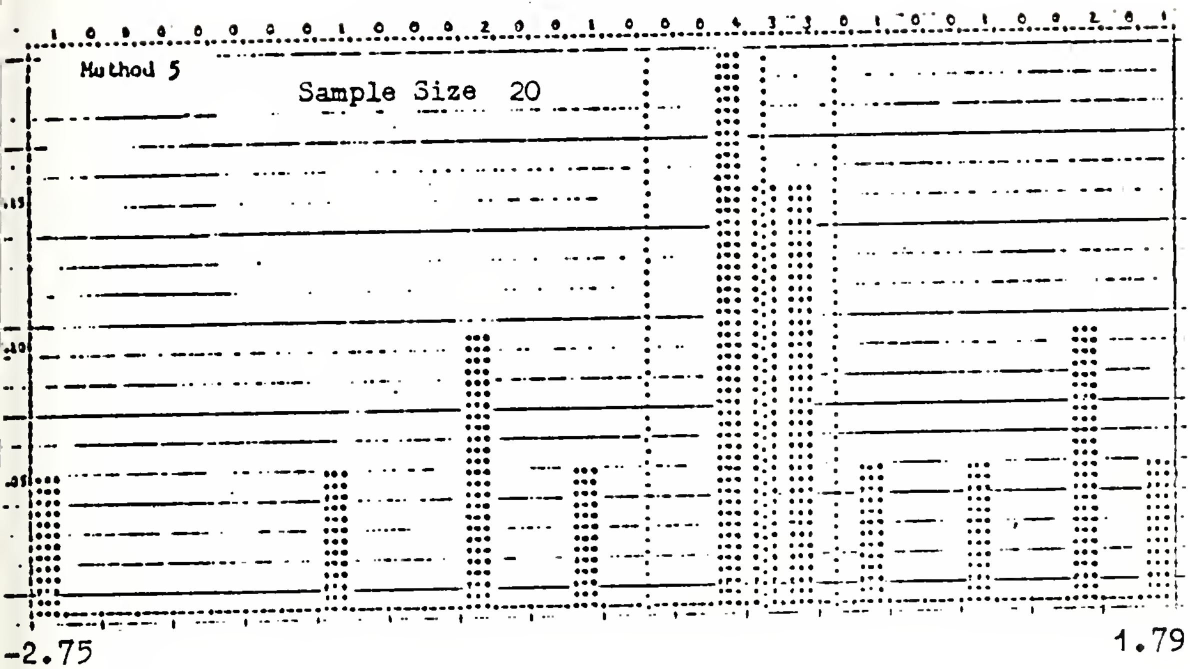
ANALYSIS OF THE RESIDUALS 155 mm

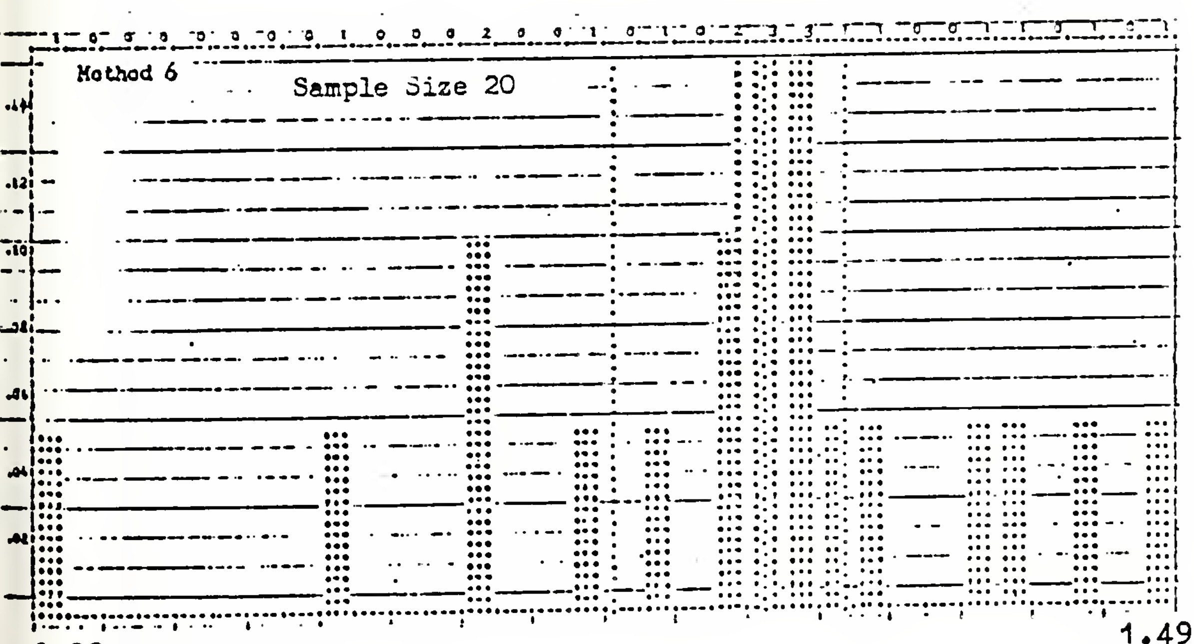




APPENDIX F

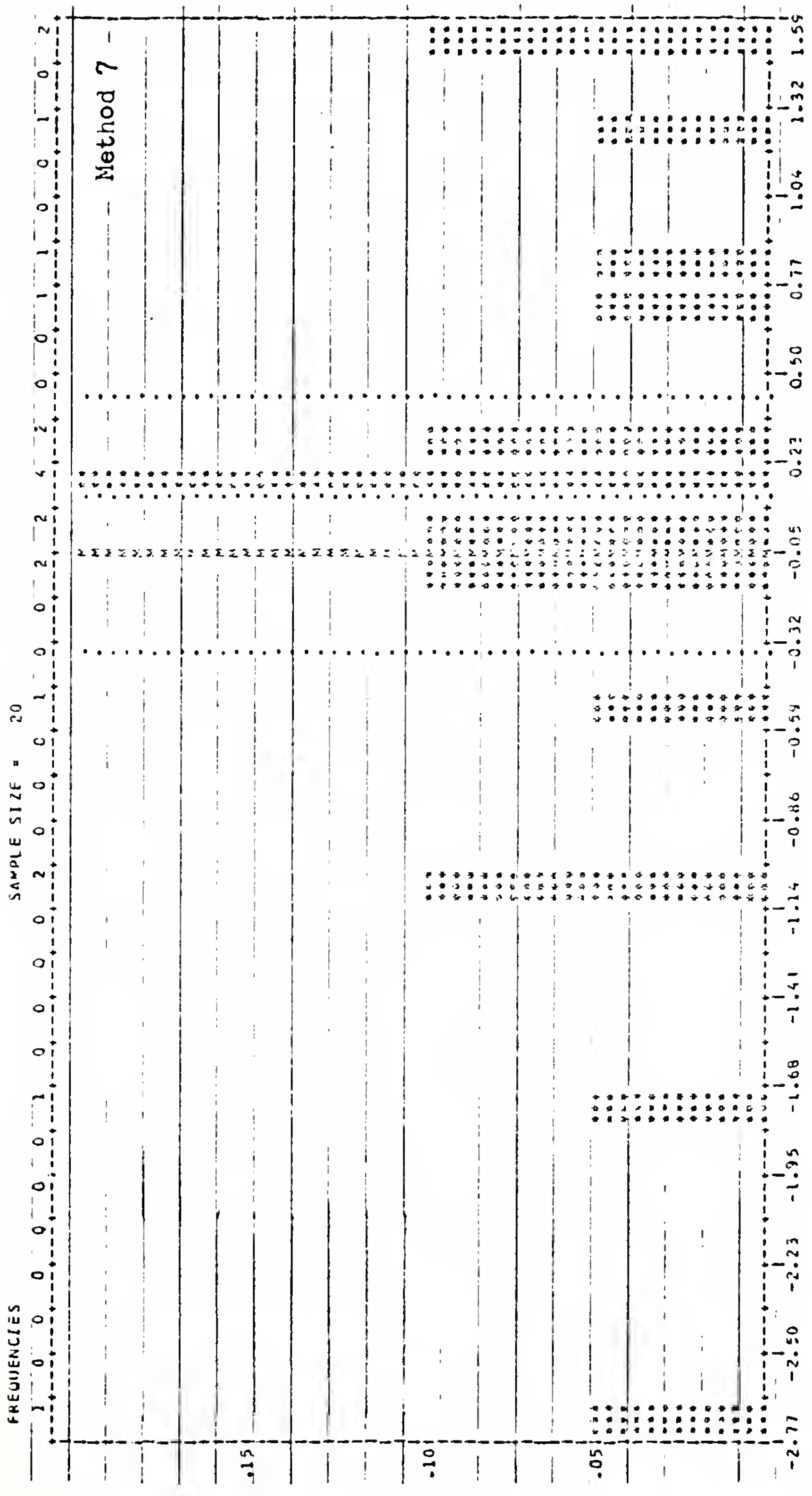
ANALYSIS OF THE RESIDUALS 155 mm





APPENDIX F

ANALYSIS OF THE RESIDUALS 155 mm

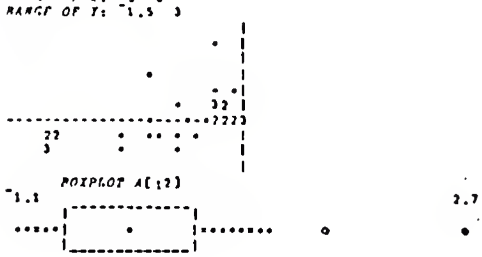


APPENDIX G

ANALYSIS OF RESIDUALS (METHCD 8)

DATA 81MM

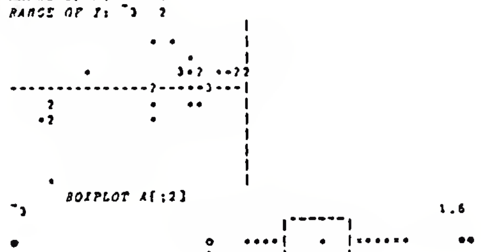
```
      REGRES
      BETA
¯0.0006387456347  0.0005057972108  0.1507461154
      K
1568.795383
      L←*BETA[3]
      L
1.162701429
      EPS
0.7974807345
      SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯1.5  3
```




```
 0.7473086         ¯0.03154771399
 1.177210450       ¯0.08336765031
 0.5708647252       0.4617836439
 0.7432371631      ¯0.03154771398
¯0.4720905831       0.1220753127
¯0.06074167496      0.5544117796
 0.01077271885     ¯0.1373044745
¯1.057400040       ¯0.5416950771
¯0.5243679402       0.1152035518
 2.722514147        0.0471677646
¯0.7781894226       0.4811403269
¯0.8153310728       0.0774097315
¯0.6273899448      ¯0.7775501055
¯0.9633533304      ¯0.6810218181 8
 1.567347274       ¯0.7047310442
¯0.6273893448      ¯0.7164727176
¯0.9633533384      ¯0.8723136306
¯0.6153318228      ¯0.5963697599
```

DATA 105MM

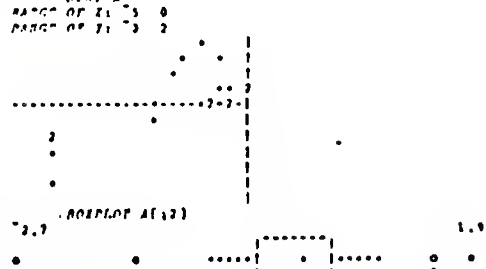
```
      REGRES
      BETA
¯0.0004621354586  0.0004014733077  ¯0.326775359
      K
2163.867717
      L←*BETA[3]
      L
0.7212457462
      EPS
0.8686269354
      SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```




```
 0.05733773479      0.533105519
¯0.6095198139       0.4476177167
 0.5747121385       0.01392775476
 0.5336668373       0.03616785783
 0.0015225563      ¯0.3281707726
¯0.5760253037      ¯0.4741591484
¯0.3567253355       0.1757500345
¯0.3567253355       0.05897573535
 1.670791206        0.1075811566
 0.7042280391       0.1625411566
¯0.3960542878       1.193311515
¯1.077260288        0.005779730 46
¯1.077260268        0.005279730 46
 1.414787651       ¯0.0417437
¯1.116525377       ¯1.057859636
 0.5172728797      ¯0.3396436 9
 0.1083660774
```

DATA 155MM

```
      REGRES
      BETA
¯0.0003771584591  0.0003305886338  0.08526835809
      K
2517.886695
      L←*BETA[3]
      L
1.083697272
      EPS
0.8323843737
      SCAT A
RANGE OF X: ¯5  0
RANGE OF Y: ¯3  2
```




```
 0.1775509571
 0.1577097738
¯1.5413664 3
 0.2409234718
¯0.0866501441
 0.9688752456
¯0.0706 7183493
¯0.0815665 1095
 1.772 2678 45
 0.0687189 9558
 0.1961144505
 1.3911107027
 0.5519217573
¯0.6118740715
¯0.9673744612
 0.1227185095
¯2.7913707 6
¯0.9073746012
 1.370575169
¯0.0415771966 8
```

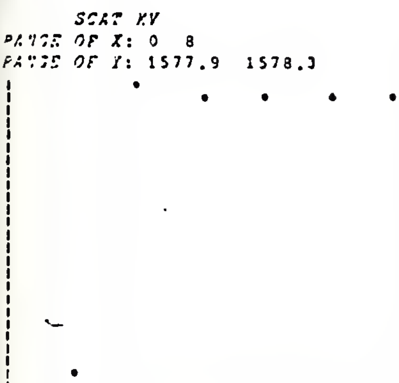
APPENDIX H

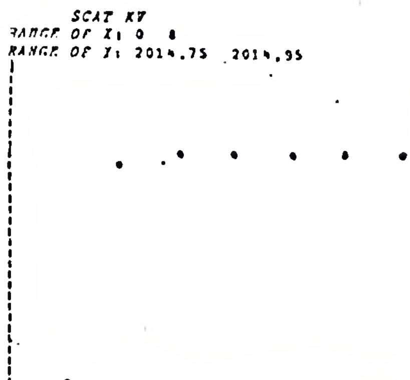
ANALYSIS OF PARAMETERS K and ϵ

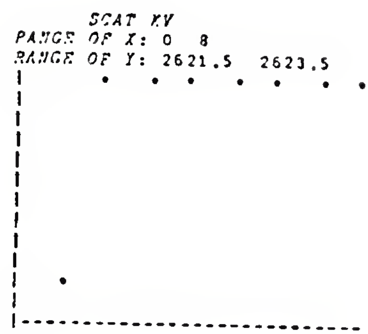
Parameter K

DATA 81MM	DATA 105MM	*DATA 155MM*

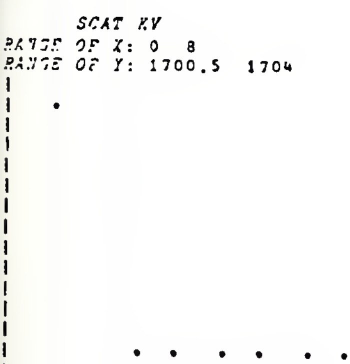
Method 5

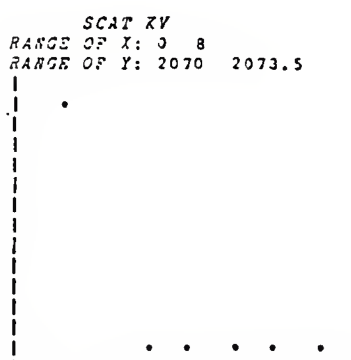


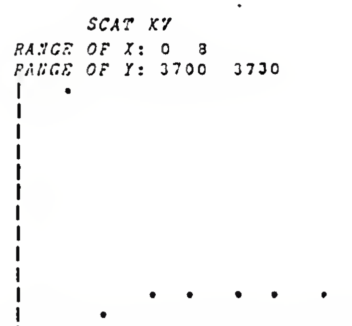




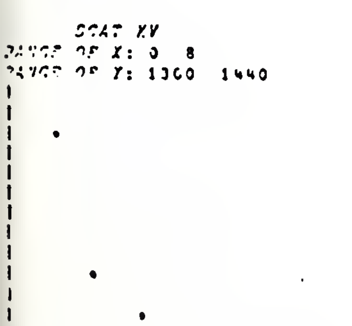
Method 6

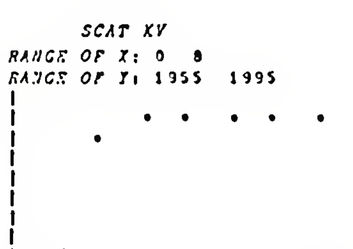


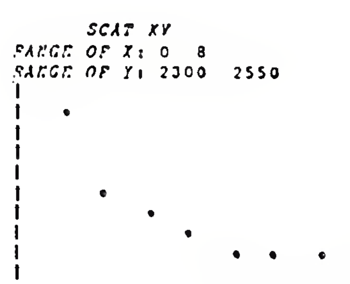




Method 7







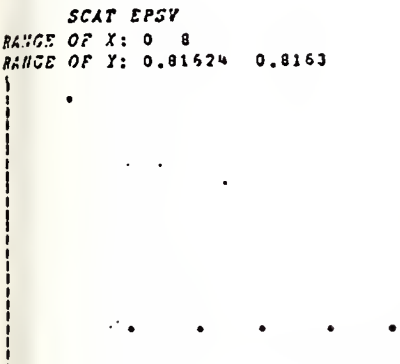
APPENDIX H

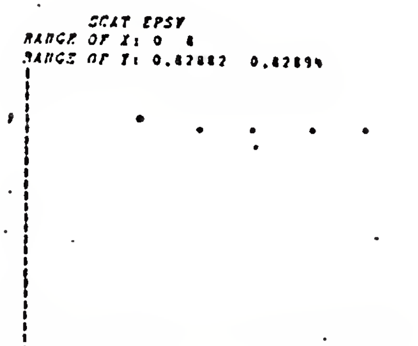
ANALYSIS OF PARAMETERS K AND ε

Parameter ε

DATA 81MM | DATA 105MM | *DATA 155MM*

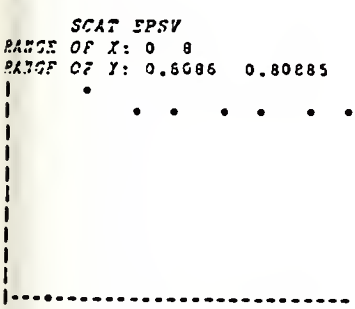
Method 5

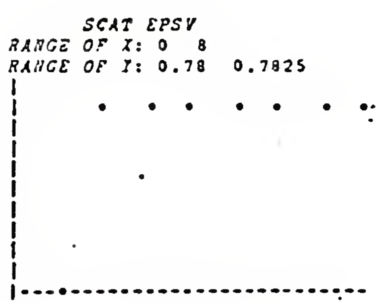


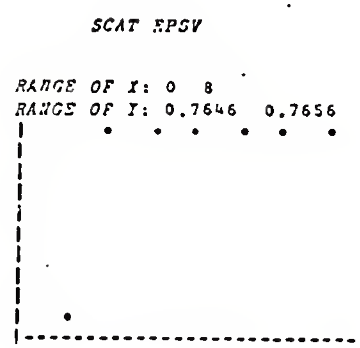


could not be plotted, change of ε-values to small.

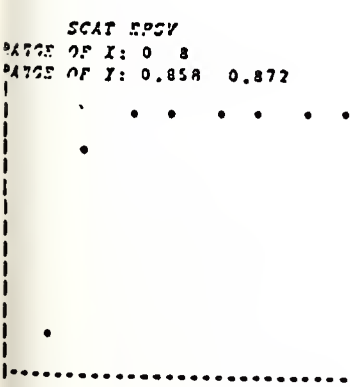
Method 6

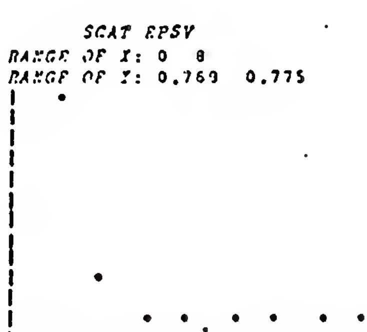


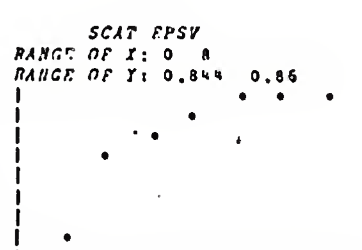




Method 7







APPENDIX H

ANALYSIS OF PARAMETERS K AND ε

The K and ε Values for Models (I) And (II)

Model	Method #	Method	81 mm K	81 mm ε	105 mm K	105 mm ε	155 mm K	155 mm ε
2.1	1	unweighted regression deletion of all zero-probabilities	4764.308	0.351	3137.117	0.727	3126.379	0.886
2.1	2	unweighted regression cluster method 2	2211.733	0.718	2217.204	0.824	3386.064	0.835
2.1	3	unweighted regression cluster method 3	2474.481	0.633	2177.751	0.825	2965.181	0.855
2.1	4	unweighted regression probability transform by Cox $y = \ln\left(\frac{S_i + \frac{1}{2}(1-p_i)}{n_i}\right)$	1591.938	0.814	2029.485	0.830	2672.144	0.828
2.1	5	unweighted iterative regression $y = \ln\left(\frac{S_i + \frac{1}{2}(1-p_i)}{n_i}\right)$	1578.287	0.816	2014.900	0.828	2623.426	0.829
2.1	6	weighted iterative regression $y = \frac{1}{SIG}\cdot\ln\left(\frac{S_i + \frac{1}{2}(1-p_i)}{n_i}\right)$ $SIG = R^2$	1700.707	0.808	2070.139	0.782	3704.303	0.765
2.1	7	weighted iterative regression $y = \frac{1}{SIG}\cdot\ln\left(\frac{S_i + \frac{1}{2}(1-p_i)}{n_i}\right)$ $SIG = \sqrt{\frac{1-p_i}{n_i p_i}}.$	1360.498	0.871	1991.151	0.769	2349.845	0.859
2.4	8	unweighted regression prob.-transform by Cox $y = \ln\left(\frac{S_i + \frac{1}{2}(1-p_i)}{n_i}\right)$	K 1566.795 L 1.163	0.792	2163.867 0.721	0.868	2517.886 1.089	0.832

APPENDIX I

APL REGRESSION PROGRAMS

NON-ITERATION METHODS

```
      ∇ REGRES
[1]     Z← 36 2 ρ1
[2]     Z[;1]←U
[3]     Z[;2]←V
[4]     ZT←⍉Z
[5]     BETA←((⌹(ZT+.×Z))+.×ZT)+.×Y
[6]     K←(¯1)÷BETA[1]
[7]     EPS←K×BETA[2]
[8]     YHAT←(BETA[1]×U)+(BETA[2]×V)
[9]     RES←Y-YHAT
[10]    A← 36 2 ρ1
[11]    A[;1]←Y
[12]    A[;2]←RES
      ∇
```

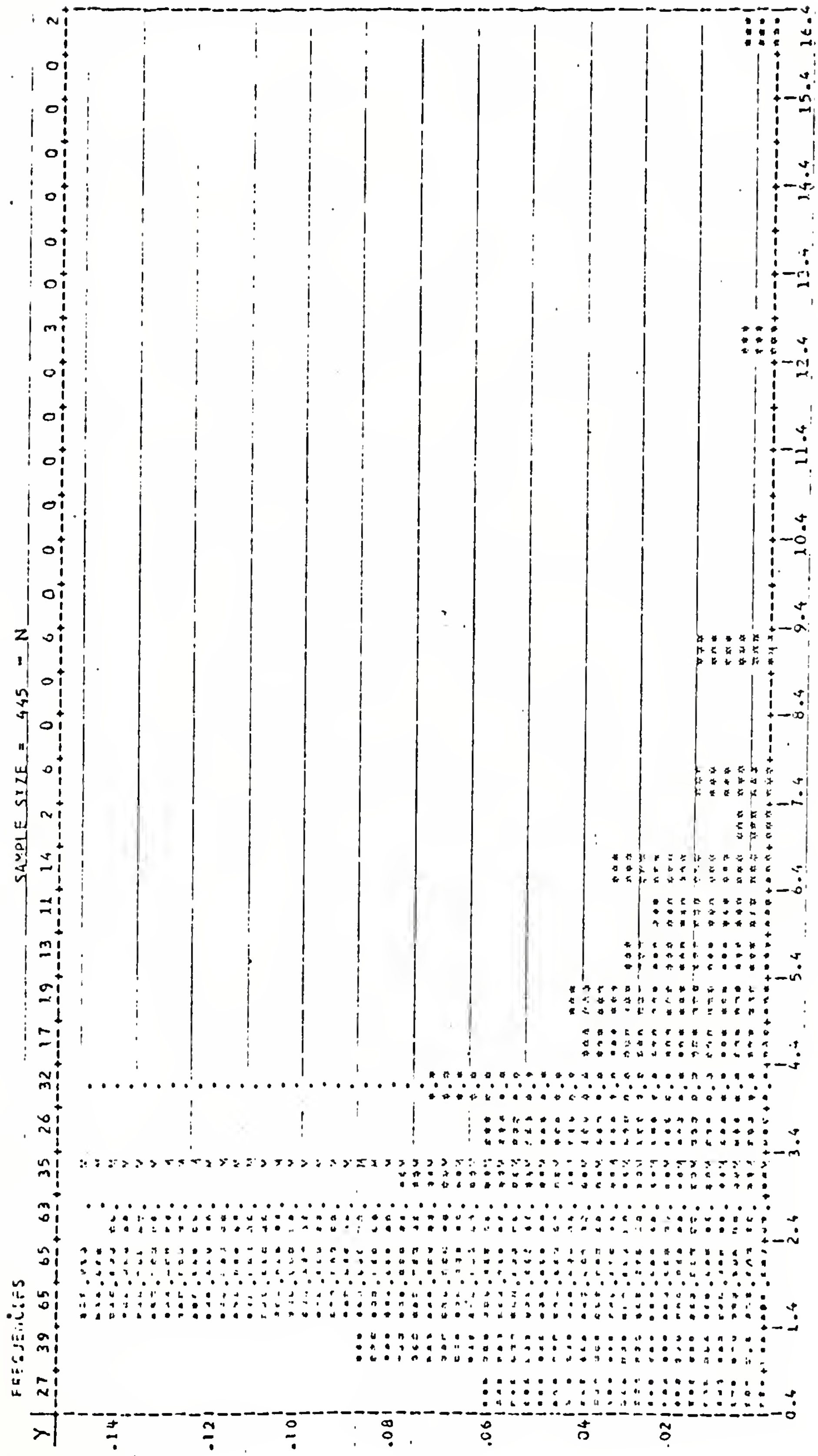
ITERATION METHODS

```
      ∇ ITER P
[1]     SIG←1
[2]     Y←(⍟((R+(0.5×(1-P)))÷N))÷SIG
[3]     U←UO÷SIG
[4]     V←VO÷SIG
[5]     Z← 36 2 ρ1
[6]     Z[;1]←U
[7]     Z[;2]←V
[8]     ZT←⍉Z
[9]     BETIT←((⌹(ZT+.×Z))+.×ZT)+.×Y
[10]    K←(¯1)÷BETIT[1]
[11]    EPS←K×BETIT[2]
[12]    YHAT←(BETIT[1]×UO)+(BETIT[2]×VO)
[13]    RES←YC-YHAT
[14]    A← 36 2 ρ1
[15]    A[;1]←YC
[16]    A[;2]←RES
      ∇
```

REMARK: SIG CHANGES IN ACCORANCE TO THE
DIFFERENT ITERATIVE METHODS.

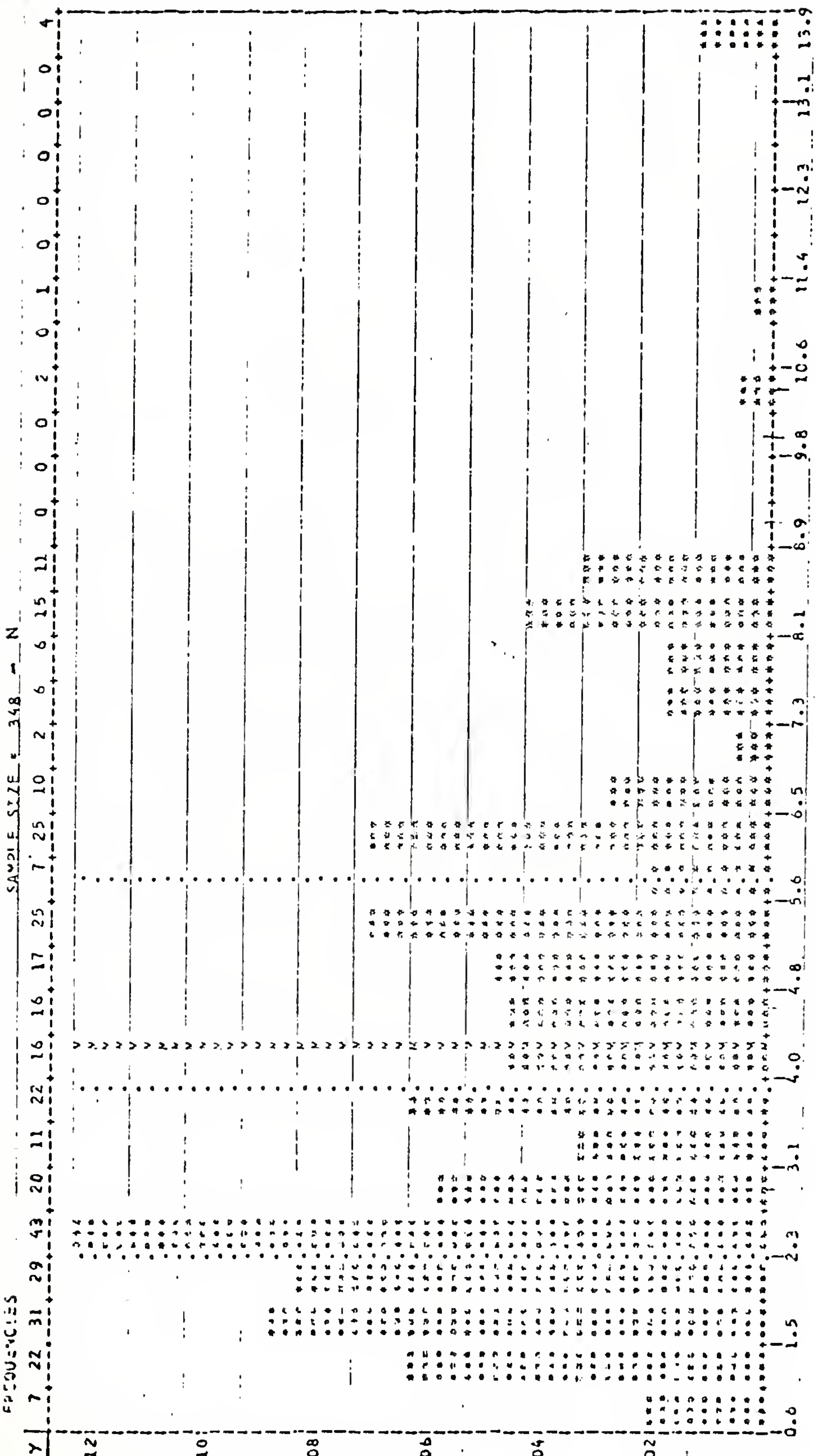
APPENDIX J

HISTOGRAMS OF SUPPRESSION INTERVALS 81 mm



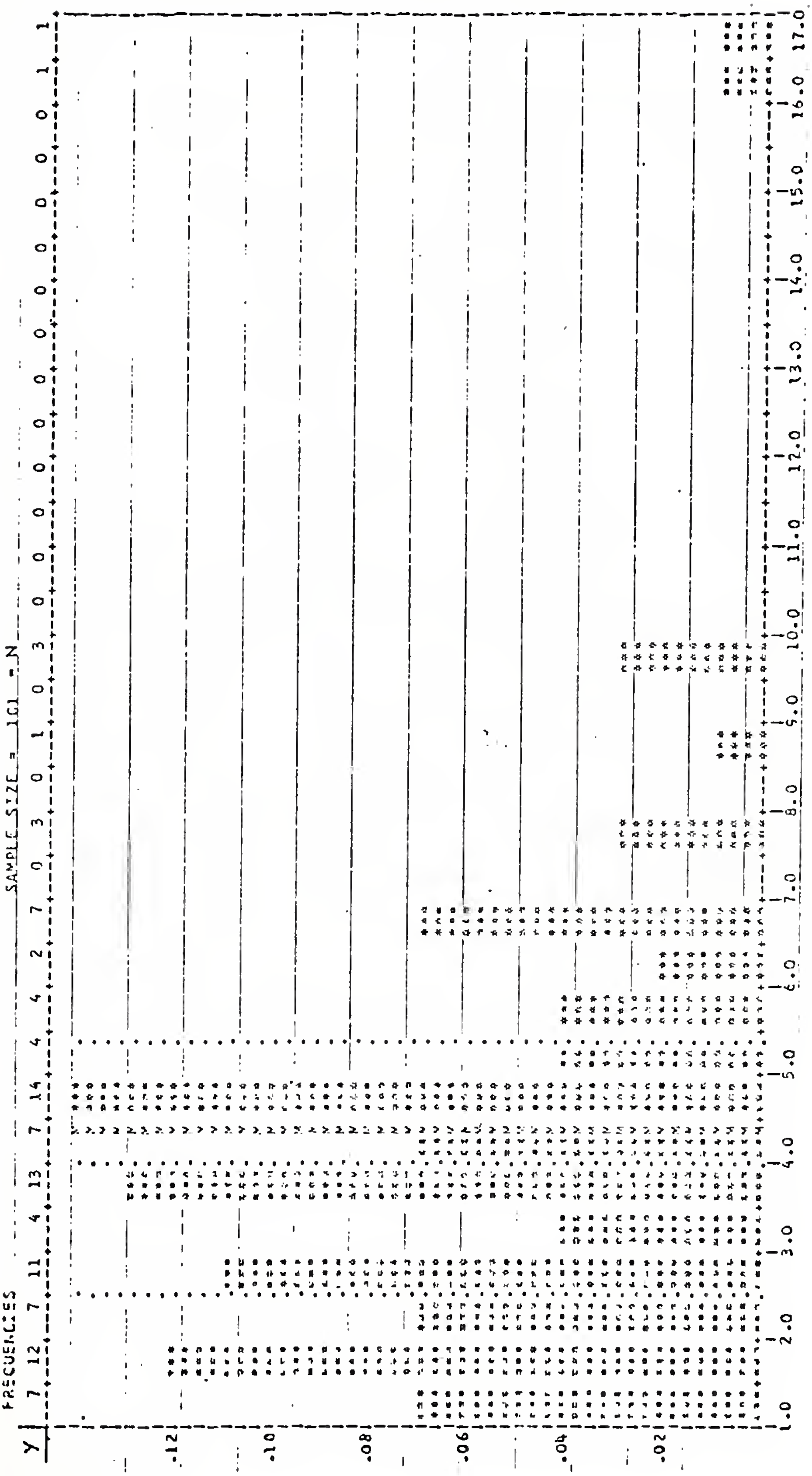
APPENDIX J

HISTOGRAMS OF SUPPRESSION INTERVALS 105 mm



APPENDIX J

HISTOGRAMS OF SUPPRESSION INTERVALS 155 mm



APPENDIX K

PROGRAMS FOR COMPUTING RATES OF RISE

Program for formula (3.54)

```
FORTRAN IV G LEVEL  21                        MAIN

                  DIMENSION TIM(500),SLA(20), RETAU(20)
                  N=348
                  READ (5,10)  (SLA(I),I=1,18)
         10       FORMAT  (F10.5)
                  READ(5,11)(TIM(I),I=1,N)
         11       FORMAT(F10.5)
                  DO 1  I=1,18
                  SUM=0.0
                  DO 2  J=1,N
                  C=EXP((SLA(I)*TIM(J)))
                  SUM=SUM+C
          2       CONTINUE
                  A=SUM/FLOAT(N)
                  B=A-1.0
                  ETAU=B/SLA(I)
                  RETAU(I)=1.0/ETAU
          1       CONTINUE
                  WRITE(6,12)(RETAU(I),I=1.18)
         12       FORMAT(1X,3F10.5)
                  STOP
                  END
```

Program for formula (3.50)

```
    ∇  LAMU
         LA←(LAMS× ((LAM-LAMS )*R ))÷(LAM*R )
         LAU←1÷((1÷LA )-(1÷LAMS ))
    ∇
```

LIST OF FIGURES

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Department of Operations Research
Naval Postgraduate School
Monterey, California 93940

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